Time: 3 Hours   **Max. Marks: 100**

General Instructions:

IMP: *Verify that you have received the question paper with the correct course, code, branch etc.*
*1. This Question paper comprises of **three Sections -A, B, & C.** It consists of Multiple Choice Questions (MCQ's) & Subjective type questions.*
2. Maximum marks for each question are indicated on right -hand side of each question.
3. Illustrate your answers with neat sketches wherever necessary.
4. Assume suitable data if necessary.
5. Preferably, write the answers in sequential order.
6. No sheet should be left blank. Any written material after a blank sheet will not be evaluated/checked.

SECTION-A   20

1. Attempt all parts:-

1-a.   If the particular integral of the PDE $(D^2 + 2DD' + D'^2)z = \cos(x + 2y)$ is   1
         $k\cos(x+2y)$ then k = _____(CO1,K2)

   (a) $\dfrac{1}{9}$

   (b) $\dfrac{1}{8}$

   (c) $-\dfrac{1}{8}$

   (d) $-\dfrac{1}{9}$

1-b.   The P.D.E $z_{xx} - 5z_{yy} = 0$ is classified as (CO1,K2)   1

   (a)   Parabolic

   (b)   Hyperbolic

   (c)   Elliptical

   (d)   None of these

1-c.   The inverse Z - Transform of F(z)=1 is ..... (CO2,K1)   1

   (a) $\delta(k)$

   (b) $u(k)$

   (c) 1

(d) None of these

1-d. $F\left(\dfrac{\partial^2 u}{\partial x^2}\right) = \ldots\ldots$is (CO2, K1)   1

(a) $-p^2\,\bar{u}$

(b) $p^2\,\bar{u}$

(c) $\bar{u}$

(d) p^2

1-e. The function $f(z) = z^2$ is (CO3,K2)   1

(a) Analytic except at origin

(b) Nowhere Analytic

(c) Analytic everywhere

(d) None of these

1-f. Imaginary part of the function z^2 is (CO3,K1)   1

(a) $x^2 - y^2$

(b) $2xy$

(c) $x^2 - y^2 x^2 + y^2$

(d) 0

1-g. $f(z) = \dfrac{z}{(z+3)(z-2)^3}$,poles and order of the poles are (CO4,K2)   1

(a) -3 ,2 and 1,3 respectively

(b) 3 , -2 and 1,3 respectively

(c) 3 ,2 and 1,1 respectively

(d) None of these

1-h. If principle part of the Laurent's series contains finite number of terms then singularity is called (CO4, K1)   1

(a) Removable

(b) Essential

(c) Pole

(d) None

1-i. A function f:X→Y is one one if (CO5,K1)   1

(a) $f(x) = f(y)$ for all x, y ∈ X

(b) $f(x) = f(y)$ implies $x = y$ for all x, y ∈ X

(c) $x = y$ implies $f(x) = f(y)$ for all x, y ∈ X

(d) None of these

1-j. Let U = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}, P = {1, 2, 5}, Q = {6, 7}. Then P ∩ Q′ is : (CO5,K2)   1

(a) P

(b) Q

(c) P'

(d) Q'

2. Attempt all parts:-

2.a. Write the solution of the heat flow in one dimension $\dfrac{\partial u}{\partial t} = c^2 \dfrac{\partial^2 u}{\partial x^2}$ which is consistent with the physical nature of the problem. (CO1,K1)　　2

2.b. Determine the $Z-$transform of $f(k) = 2^k$ for $k \geq 0$. (CO2,K3)　　2

2.c. Find the fixed points under the transformation $w = \dfrac{2z-5}{z+4}$. (CO3,K3)　　2

2.d. Evaluate: $\displaystyle\int_0^{(1+i)} (x^2 - iy)dz$ along the path $y = x^2$. (CO4,K3)　　2

2.e. How many words can be formed by using all letters of the word "CHIRAG"? (CO5,K1)　　2

SECTION-B　　30

3. Answer any <u>five</u> of the following:-

3-a. Solve: $(D - D' - 1)(D - D' - 2)z = \sin(2x + 3y)$. (CO1,K3)　　6

3-b. Solve: $r + s - 2t = \sqrt{(2x + y)}$. (CO1,K3)　　6

3-c. Find the $Z-$transform of (i) $f(k) = k^2$ (ii) $f(k) = k(k-1)$ (CO2,K3)　　6

3-d. Find the Fourier transform of the following function: (CO2,K3)

$$F(t) = \begin{cases} t, & |t| < a \\ 0, & |t| > a \end{cases}$$

6

3.e. Show that $e^x \cos y$ is a harmonic function. Also find the corresponding analytic function whose real part is given. (CO3,K2)　　6

3.f. Evaluate $\displaystyle\oint_C \dfrac{e^z}{(z-1)(z-4)} dz$, where C is the circle $|z| = 2$. (CO4,K3)　　6

3.g. Statements: All men are vertebrates. Some mammals are vertebrates.
Conclusions:
All men are mammals.
All mammals are men.
Some vertebrates are mammals.
All vertebrates are men.
Justify your answer. (CO5,K2)　　6

SECTION-C　　50

4. Answer any <u>one</u> of the following:-

4-a. Use the method of separation of variables to solve the equation:

$$\frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial y} + 2u. \text{ (CO1,K3)}$$

10

4-b. A string is stretched and fastend to two points L apart. Motion is started by displacing the string in the form $y(x,t) = A\sin \dfrac{\pi x}{L}$ from which it is released at time t=0. Show that the displacement of any point at a displacement of any point at a distance x from one end at time t is given by:
$y(x,t) = A\sin \dfrac{\pi x}{L} \cos \dfrac{\pi ct}{L}$ (CO1,K3)   **10**

5. Answer any <u>one</u> of the following:-

5-a. Find the Fourier cosine transform of the function – (CO2,K3)   **10**

$f(x) = \begin{cases} \cos x, & 0 < x < a \\ 0, & x > a \end{cases}$

5-b. Using Z−transform, solve the equation: (CO2,K3)   **10**

$y_{k+1} + \dfrac{1}{4}y_k = \left(\dfrac{1}{4}\right)^k, \ k \geq 0, y(0) = 0.$

6. Answer any <u>one</u> of the following:-

6-a. Examine the nature of the function $f(z) = \dfrac{x^3 y\,(y - i\,x)}{(x^6 + y^2)}, z \neq 0$ and $f(0) = 0$ **10**

in the region including the origin. (CO3,K3)

6-b. If $u - v = (x-y)(x^2 + 4xy + y^2)$ and $f(z) = u + iv$ is an analytic function of $z = x + iy$, find $f(z)$ in terms of z. (CO3,K3)   **10**

7. Answer any <u>one</u> of the following:-

7-a. Evaluate by using Cauchy's Residue theorem: $\displaystyle\int_C \dfrac{(4 - 3z)}{z(z-1)(z-2)}\,dz$,   **10**

where C is the circle $|z| = 3/2$. (CO4,K3)

7-b. Expand $\dfrac{1}{(z^2 - 3z + 2)}$ in the region (CO4,K3)   **10**

(i) $|z| < 1$

(ii) $1 < |z| < 2$

8. Answer any <u>one</u> of the following:-

8-a. Answer the questions based on the following table. (CO5,K2)   **10**
The table gives the production of major agricultural products in Million Tonnes (MT).

   1. Which agro product witnessed the highest growth rate in production from 1997 to 2000.
   2. Pulses production in 1998 is what percent of the total production of rice in the given 4-year period?
   3. By what percent is the average wheat production more than the average sugar cane production for the given 4-year period?
   4. What is the simple annual growth rate of wheat from 1997-2000 ?
   5. What would be the actual production of wheat in 2001 if the growth in 2001

is the same as the average growth for the period?

Year	Wheat	Rice	Sugar cane	Pulses
1997	100	91	15	71
1998	120	88	18	75
1999	125	97	21	79
2000	131	107	25	88

8-b. The probability that machine *A* will be performing an usual function in 5 years 10

time is $\frac{1}{4}$, while the probability that machine *B* will be operating usually at the

end of the same period is $\frac{1}{3}$. Find the probability in the following cases that in 5 years time: (CO5,K3)

i. Both machines will be performing an usual function
ii. Neither will be operating
iii. Only machine B will be operating
iv. At least one of the machines will be operating