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NOIDA INSTITUTE OF ENGINEERING AND TECHNOLOGY, GREATER NOIDA**(An Autonomous Institute Affiliated to AKTU, Lucknow)****B.Tech.****SEM: III- THEORY EXAMINATION (2024-2025)****Subject DISCRETE STRUCTURES****Time: 3 Hours****Max. Marks:100****General Instructions:****IMP:** Verify that you have received question paper with correct course, code, branch etc.

1. This Question paper comprises of three Sections -A, B, & C. It consists of Multiple Choice Questions (MCQ's) & Subjective type questions.
2. Maximum marks for each question are indicated on right hand side of each question.
3. Illustrate your answers with neat sketches wherever necessary.
4. Assume suitable data if necessary.
5. Preferably, write the answers in sequential order.
6. No sheet should be left blank. Any written material after a blank sheet will not be evaluated/checked.

SECTION – A**20**

1. Attempt all parts:-

- 1-a. Which among the following can be taken as the discrete object : (CO1,K2) 1
- (a) People
 - (b) Rational Numbers
 - (c) Integers
 - (d) All the mentioned
- 1-b. The function (gof) is _____, if the function f and g are onto function : 1
(CO1,K2)
- (a) Onto function
 - (b) into function
 - (c) one-to-one function
 - (d) one-to-many function
- 1-c. Select incorrect statemen if X and Y are two nonempty relations on the set S. 1
(CO2,K3)
- (a) If X and Y are transitive, then the intersection of X and Y is also transitive commutative
 - (b) If X and Y are reflexive, then the intersection of X and Y is also reflexive
 - (c) If X and Y are symmetric, then the union of X and Y is not symmetric
 - (d) If X and Y are transitive, then the union of X and Y is not transitive

- 1-d. A semigroup S under binary operation $*$ that has an identity is called _____ (CO2,K2) 1
- (a) multiplicative identity
 - (b) monoid
 - (c) subgroup
 - (d) homomorphism
- 1-e. Counting the elements in the group G determines the _____ of the group. (CO3,K2) 1
- (a) number
 - (b) elements
 - (c) order
 - (d) pair
- 1-f. Due to the _____ nature of partial orders, in Hasse diagrams, some edges between vertices are deleted. (CO3,K3) 1
- (a) Transitivity
 - (b) Reflexivity
 - (c) Associativity
 - (d) Antisymmetric
- 1-g. The following identities are valid if L is a bounded lattice. (CO4,K3) 1
- (a) $a \vee 1 = 1$
 - (b) $a \wedge 1 = a$
 - (c) $a \vee 0 = a$
 - (d) all of the above
- 1-h. Contrapositive of $p \rightarrow q$ is the proposition _____. (CO4,K1) 1
- (a) $\sim p \rightarrow \sim q$
 - (b) $\sim q \rightarrow \sim p$
 - (c) $q \rightarrow \sim p$
 - (d) $\sim q \rightarrow p$
- 1-i. In a connected undirected graph with n vertices and e edges, what is the minimum number of edges required? (CO5,K1) 1
- (a) $n - 1$
 - (b) $e - n$
 - (c) $n + 1$
 - (d) n^2
- 1-j. Select graph which is non-planar : (CO5,K1) 1
- a) K_4
 - b) K_5
 - c) star graph
 - d) path graph

2. Attempt all parts:-

- | | | |
|------|---|---|
| 2.a. | Explain the concept of composition of relations and inverse relations.(CO1,K2) | 2 |
| 2.b. | Define algebraic structure. List the properties of a semigroup.(CO2,K2) | 2 |
| 2.c. | Define a partially ordered set. Give an example of Poset. (CO3,K2) | 2 |
| 2.d. | Define proposition and compound proposition with examples. (CO4,K2) | 2 |
| 2.e. | Define a graph. Explain the differences between directed and undirected graphs.(CO5,K2) | 2 |

SECTION – B

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3. Answer any five of the following-

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|------|--|---|
| 3-a. | Provide an example of an equivalence relation and a partial order relation.(CO1,K3) | 6 |
| 3-b. | Prove or disprove the following statements about sets:
(i) $A - (B \cap C) = (A - B) \cap (A - C)$ for all sets A, B, C.
(ii) $A - (B \cap C) = (A - B) \cup (A - C)$ for all sets A, B, C. (CO1,K3) | 6 |
| 3-c. | Explain group and its properties with an example. (CO2,K2) | 6 |
| 3-d. | Describe applications of Algebraic structures.(CO2,K2) | 6 |
| 3-e. | Draw the lattice of all divisors of 36 and explain whether it is distributive or not.(CO3,K3) | 6 |
| 3-f. | Discuss the difference between tautology, contradiction, and contingency with examples.(CO4,K2) | 6 |
| 3-g. | Explain chromatic number with example. Mention real life applications.(CO5,K2) | 6 |

SECTION – C

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4. Answer any one of the following-

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|------|--|----|
| 4-a. | Draw the directed graph for the relation R defined by the matrix | 10 |
|------|--|----|

$$M_R = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 \end{bmatrix}$$

Also determine the properties of relation R .(CO1,K3)

- | | | |
|------|--|----|
| 4-b. | Let $f: \mathbb{N} \rightarrow \mathbb{Z}$ be defined by the two-part rule | 10 |
|------|--|----|

$$f(n) = \begin{cases} n/2, & \text{if } n \text{ is even} \\ -(n+1)/2, & \text{if } n \text{ is odd} \end{cases}$$

Determine whether f is one-to-one. Determine whether f is onto $\mathbb{Z}$.(CO1,K4)

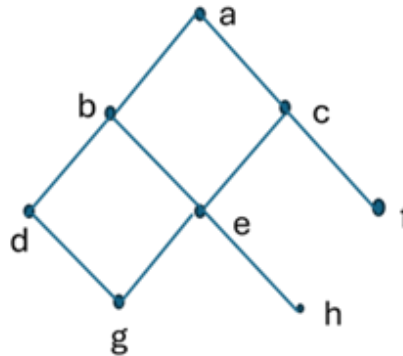
5. Answer any one of the following-

- | | | |
|------|---|----|
| 5-a. | Determine whether the following groups are Abelian or non-Abelian. Justify your answers:
(i) $(\mathbb{Z}_5, +_5)$
(ii) S_3 (Symmetric group on 3 elements)
(iii) $(\mathbb{R}^*, \cdot)$ (nonzero real numbers under multiplication) (CO2,K4) | 10 |
| 5-b. | Differentiate between homomorphism and Isomorphism of groups with examples. (CO2,K3) | 10 |

6. Answer any one of the following-

6-a. Consider the poset P with Hasse diagram as shown below:

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(i) List the maximal members and minimal members of P.

(ii) Is there a largest member of P? If so, what is it? Is there a smallest member of P? If so, what is it?

(iii) Determine: $\text{LUB}\{b, c\}$, $\text{GLB}\{b, c\}$, $\text{LUB}\{b, f\}$, $\text{GLB}\{b, f\}$, $\text{LUB}\{g, c\}$, and $\text{GLB}\{g, c\}$. (CO3,K4)

6-b. Define the bounded lattice and distributive lattice. Give an example of each. Let L be a bounded distributive lattice then show that if a complement exists in L it is unique. (CO3,K4)

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7. Answer any one of the following-

7-a. (i) Prove or disprove that $(p \rightarrow q) \rightarrow r$ and $p \rightarrow (q \rightarrow r)$ are equivalent.

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(ii) Prove that $(\neg q \wedge (p \rightarrow q)) \rightarrow \neg p$ is a tautology using logical equivalences. (CO4,K4)

7-b. Explain how inference rules can be used to prove logical arguments and compare this method with truth tables.

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Using truth table prove that:

$P \leftrightarrow Q \equiv (P \rightarrow Q) \wedge (Q \rightarrow P)$. (CO4,K4)

8. Answer any one of the following-

8-a. State and prove Euler's theorem for connected graphs. Use an example to verify the theorem. (CO5,K4)

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8-b. Define complete graphs and planar graphs. Use Kuratowski's theorem to determine whether the graph K_5 is planar. (CO5,K4)

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