Printed page: 04 **Subject Code: BMICSE0306**

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NOIDA INSTITUTE OF ENGINEERING AND TECHNOLOGY, GREATER NOIDA

(An Autonomous Institute Affiliated to AKTU, Lucknow)

M.Tech(Integrated)

SEM: III- THEORY EXAMINATION (2024-2025)

Subject DISCRETE STRUCTURES

Time: 3 Hours **Max. Marks:100**

General Instructions:

IMP: Verify that you have received question paper with correct course, code, branch etc.

1. *This Question paper comprises of three Sections -A, B, & C. It consists of Multiple Choice Questions (MCQ's) & Subjective type questions.*
2. *Maximum marks for each question are indicated on right hand side of each question.*
3. *Illustrate your answers with neat sketches wherever necessary.*
4. *Assume suitable data if necessary.*
5. *Preferably, write the answers in sequential order.*
6. *No sheet should be left blank. Any written material after a blank sheet will not be evaluated/checked.*

SECTION – A 20

1. Attempt all parts:-

1-a. Which among the following can be taken as the discrete object : (CO1,K2) 1

(a) People
(b) Rational Numbers
(c) Integers
(d) All the mentioned

1-b. The function (gof) is __________ , if the function f and g are onto function : (CO1,K2) 1

(a) Onto function
(b) into function
(c) one-to-one function
(d) one-to-many function

1-c. Select incorrect statemen if *X* and *Y* are two nonempty relations on the set *S*. (CO2,K3) 1

(a) If X and Y are transitive, then the intersection of X and Y is also transitive commutative
(b) If X and Y are reflexive, then the intersection of X and Y is also reflexive
(c) If X and Y are symmetric, then the union of X and Y is not symmetric
(d) If X and Y are transitive, then the union of X and Y is not transitive

1-d. A semigroup S under binary operation * that has an identity is called __________(CO2,K2) 1

(a) multiplicative identity
(b) monoid
(c) subgroup
(d) homomorphism

1-e. Counting the elements in the group G determines the ____ of the group.(CO3,K2) 1

(a) number
(b) elements
(c) order
(d) pair

1-f. Due to the _____ nature of partial orders, in Hasse diagrams, some edges between vertices are deleted. (CO3,K3) 1

(a) Transitivity
(b) Reflexivity
(c) Associativity
(d) Antisymmetric

1-g. The following identities are valid if *L* is a bounded lattice.(CO4,K3) 1

(a) $a \vee 1 = 1$
(b) $a \wedge 1 = a$
(c) $a \vee 0 = a$
(d) all of the above

1-h. Contrapositive of $p \rightarrow q$ is the proposition ____: .(CO4,K1) 1

(a) $\sim p \rightarrow \sim q$
(b) $\sim q \rightarrow \sim p$
(c) $q \rightarrow \sim p$
(d) $\sim q \rightarrow p$

1-i. In a connected undirected graph with n vertices and e edges, what is the minimum number of edges required? (CO5,K1) 1

(a) $n - 1$
(b) e-n
(c) n + 1
(d) n^2

1-j. Select graph which is non-planar : (CO5,K1) 1

a) K_4
b) K_5
c) star graph
d) path graph

2. Attempt all parts:-

2.a.	Explain the concept of composition of relations and inverse relations.(CO1,K2)	2
2.b.	Define algebraic structure. List the properties of a semigroup.(CO2,K2)	2
2.c.	Define a partially ordered set. Give an example of Poset. (CO3,K2)	2
2.d.	Define proposition and compound proposition with examples. (CO4,K2)	2
2.e.	Define a graph. Explain the differences between directed and undirected graphs.(CO5,K2)	2

SECTION – B 30

3. Answer any <u>five</u> of the following-

3-a.	Provide an example of an equivalence relation and a partial order relation.(CO1,K3)	6
3-b.	Prove or disprove the following statements about sets: (i) A - (B ∩ C) = (A - B) ∩ (A - C) for all sets A, B, C. (ii) A - (B ∩ C) = (A - B) ∪ (A - C) for all sets A, B, C. (CO1,K3)	6
3-c.	Explain group and its properties with an example. (CO2,K2)	6
3-d.	Describe applications of Algebraic structures.(CO2,K2)	6
3-e.	Draw the lattice of all divisors of 36 and explain whether it is distributive or not.(CO3,K3)	6
3-f.	Discuss the difference between tautology, contradiction, and contingency with examples.(CO4,K2)	6
3-g.	Explain chromatic number with example. Mention real life applications.(CO5,K2)	6

SECTION – C 50

4. Answer any <u>one</u> of the following-

4-a. Draw the directed graph for the relation R defined by the matrix — 10

$$M_R = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 \end{bmatrix}$$

Also determine the properties of relation R.(CO1,K3)

4-b. Let f: $\mathbb{N} \rightarrow \mathbb{Z}$ be defined by the two-part rule — 10

$$f(n) = \begin{cases} n/2, & \text{if } n \text{ is even} \\ -(n+1)/2, & \text{if } n \text{ is odd} \end{cases}$$

Determine whether f is one-to-one. Determine whether f is onto $\mathbb{Z}$.(CO1,K4)

5. Answer any <u>one</u> of the following-

5-a. Determine whether the following groups are Abelian or non-Abelian. Justify your answers: — 10

(i) $(Z_5, +_5)$

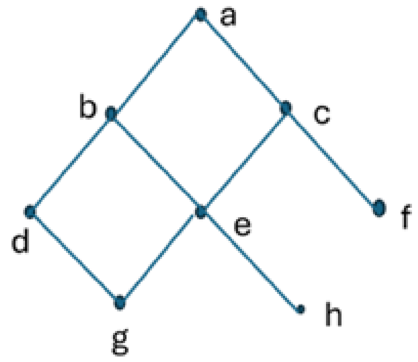
(ii) S_3 (Symmetric group on 3 elements)

(iii) $(R^*, \cdot)$ (nonzero real numbers under multiplication) (CO2,K4)

5-b. Differentiate between homomorphism and Isomorphism of groups with examples. (CO2,K3) — 10

6. Answer any <u>one</u> of the following-

6-a. Consider the poset P with Hasse diagram as shown below: 10



(i) List the maximal members and minimal members of P.

(ii) Is there a largest member of P? If so, what is it? Is there a smallest member of P? If so, what is it?

(iii) Determine: LUB{b, c}, GLB{b, c}, LUB{b, f}, GLB{b, f}, LUB{g, c}, and GLB{g, c}. (CO3,K4)

6-b. Define the bounded lattice and distributive lattice. Give an example of each. Let L be a bounded distributive lattice then show that if a complement exists in L it is unique.(CO3,K4) 10

7. Answer any <u>one</u> of the following-

7-a. (i) Prove or disprove that $(p \rightarrow q) \rightarrow r$ and $p \rightarrow (q \rightarrow r)$ are equivalent. 10

(ii) Prove that $(\neg q \wedge (p \rightarrow q)) \rightarrow \neg p$ is a tautology using logical equivalences. (CO4,K4)

7-b. Explain how inference rules can be used to prove logical arguments and compare this method with truth tables. 10

Using truth table prove that:

$P \leftrightarrow Q \equiv (P \rightarrow Q) \wedge (Q \rightarrow P)$. (CO4,K4)

8. Answer any <u>one</u> of the following-

8-a. State and prove Euler's theorem for connected graphs. Use an example to verify the theorem. (CO5,K4) 10

8-b. Define complete graphs and planar graphs. Use Kuratowski's theorem to determine whether the graph K_5 is planar. (CO5,K4) 10