Printed page: …. **Subject Code: AMIAS0103**

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NOIDA INSTITUTE OF ENGINEERING AND TECHNOLOGY, GREATER NOIDA

(An Autonomous Institute Affiliated to AKTU, Lucknow)

M.**Tech**(Integrated)

SEM: I - THEORY EXAMINATION (2022-2023)

Subject: Engineering Mathematics-I

Time: 3 Hours **Max. Marks:100**

General Instructions:

IMP: Verify that you have received question paper with correct course, code, branch etc.

1. *This Question paper comprises of three Sections -A, B, & C. It consists of Multiple Choice Questions (MCQ's) & Subjective type questions.*
2. *Maximum marks for each question are indicated on right hand side of each question.*
3. *Illustrate your answers with neat sketches wherever necessary.*
4. *Assume suitable data if necessary.*
5. *Preferably, write the answers in sequential order.*
6. *No sheet should be left blank. Any written material after a blank sheet will not be evaluated/checked.*

SECTION – A 20

1. Attempt all parts:-

1-a. If '0' is a characteristic root of a matrix if and only if the matrix is (CO1) 1

(a) Identity

(b) Singular

(c) Non-singular

(d) None of these

1-b. If a matrix A is called Idempotent then (CO1) 1

(a) $A^2 = O$

(b) $A^2 = I$

(c) $A^2 = A$

(d) None of these

1-c. The nth derivative of $\sin 3x$ is (CO2) 1

(a) $cos3x$

(b) $3^n\ sin3x$

(c) $3^n \sin(\frac{n\pi}{2} + 3x)$

(d) $3^n \cos(\frac{n\pi}{2} + 3x)$

1-d. $u = \frac{x^4+y^4}{x-y}$ is a homogeneous function of order (CO2) 1

(a) 4

(b) 3

(c) 2

(d) 1

1-e. If any function is maxima then it satisfied the condition (CO3) 1

(a) $rt - s^2 < 0$ and $r < 0$

(b) $rt - s^2 < 0$ and $r > 0$

(c) $rt - s^2 > 0$ and $r < 0$

(d) $rt - s^2 > 0$ and $r > 0$

1-f. If $x = r\,cos\theta, y = r\,sin\theta$, then the value of the Jacobian $\frac{\partial(x,y)}{\partial(r,\theta)}$ is (CO3) 1

(a) $r\,sin\theta$

(b) r

(c) $r\,cos\theta$

(d) 0

1-g. The value of $\int_0^{\pi}\int_0^{a(1+cos\theta)} drd\theta$ is (CO4) 1

(a) $a\pi$

(b) a

(c) π

(d) 0

1-h. If $x = r\,cos\theta, y = r\,sin\theta$, then $F(x,y)dxdy$ in terms of $drd\theta$ is (CO4) 1

(a) $F(r\,cos\theta, r\,sin\theta)drd\theta$

(b) $F(r\,cos\theta, r\,sin\theta)rdrd\theta$

(c) $F(r\,cos\theta, r\,sin\theta)r^2drd\theta$

(d) None of these

1-i. If a number increases from 20 to 28, then the increasing percentage is (CO5) 1

(a) 8%

(b) 20.57%

(c) 28.57%

(d) None of these

1-j. If the mean of 4 observations is 20, when a constant C is added to each observation, then mean becomes 22. The value of C is (CO5) 1

(a) 30

(b) 22

(c) 28

(d) None of these

2. Attempt all parts:-

2.a. Test whether the given equation is unique solution or infinite solution : 2

$x + 3y + 2z = 0, 2x - y + 3z = 0, 3x - 5y + 4z = 0.$ (CO1)

2.b. If $u = \left(x^{\frac{1}{4}} + y^{\frac{1}{4}}\right)\left(x^{\frac{1}{7}} - y^{\frac{1}{7}}\right)$, apply Euler's theorem to find the value of $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}$. (CO2) 2

2.c. If the base radius and height of a cone are measured as 4 and 8 inches with a possible error of 0.04 and 0.08 inches respectively, calculate the percentage (%) error in calculating volume of the cone. (CO3) 2

2.d. Evaluate $\int_0^{\frac{\pi}{2}} \sqrt{tanx}\, dx$. (CO4) 2

2.e. Rajesh purchased 35 kg of rice at the rate of Rs. 9.50 per kg and 30 kg at the rate of 10.50 per kg. He mixed the two. Approximately, at what price per kg should he sell the mixture to make 35% profit in the transaction? (CO5) 2

SECTION – B

30

3. Answer any <u>five</u> of the following-

3-a. Test the consistency of following equations, and if possible, find the solution: 6

$4x - 2y + 6z = 8, x + y - 3z = -1, 15x - 3y + 9z = 21.$ (CO1)

3-b. Reduce the following matrix A to the normal form and find its rank, where 6

$$A = \begin{bmatrix} 1 & 2 & -1 & 4 \\ 2 & 4 & 3 & 4 \\ 1 & 2 & 3 & 4 \\ -1 & -2 & 6 & -7 \end{bmatrix}.$$ (CO1)

3-c. If $u = tan^{-1}\frac{x^3+y^3}{x-y}$, prove that $x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2} = sin4u - sin2u$. (CO2) 6

3-d. Trace the following curve $x^3 + y^3 = 3axy$. (CO2) 6

3-e. Show that the function $F(x, y) = x^3 + y^3 - 63(x + y) + 12xy$ is maximum at (-7,-7) and minimum at (3,3) . (CO3) 6

3-f. Evaluate $\iint_R (x + y)^2 dxdy$, where R is the parallelogram in the xy-plane with vertices (1,0), (3,1),(2,2), (0,1), using the transformation $u = x + y$ and $v = x - 2y$. (CO4) 6

3-g. Arun sells an object to Benny at a profit of 15%, Benny sells that object to Chandan for Rs. 1012 and makes a profit of 10%. At what cost did Arun purchase the object? (CO5) 6

SECTION – C

50

4. Answer any one of the following-

4-a. Find Eigen values and Eigen vectors of the matrix $\begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$. (CO1) 10

4-b. Verify Cayley-Hamilton theorem for the matrix $\begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$. Hence find A^{-1}. Also evaluate $A^6 - 6A^5 + 9A^4 - 2A^3 - 12A^2 + 23A - 9I$. (CO1) 10

5. Answer any one of the following-

5-a. Find $y_n(0)$, if $y = \sin(a\ sin^{-1}x)$. (CO2) 10

5-b. Explain total derivative of a function. If $u = u\left(\frac{y-x}{yx}, \frac{z-x}{zx}\right)$, then prove that $x^2\frac{\partial u}{\partial x} + y^2\frac{\partial u}{\partial y} + z^2\frac{\partial u}{\partial z} = 0$. (CO2) 10

6. Answer any one of the following-

6-a. Expand $e^x sin\ y$ in powers of x and y as for as terms of third degree. (CO3) 10

6-b. If $u = x + y + z, v = x^2 + y^2 + z^2, w = x^3 + y^3 + z^3 - 3xyz$, prove that u, v, w are not independent and hence find the relation between them. (CO3) 10

7. Answer any one of the following-

7-a. Evaluate the following integral by changing the order of integration 10

$\int_0^1 \int_{y^2}^{2-y} xydxdy$. (CO4)

7-b. Find the volume of the tetrahedron bounded by the plane $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$ and the coordinate planes using Dirichlet's integral. (CO4) 10

8. Answer any one of the following-

8-a. (i) Simplify $\frac{1}{3+\sqrt{7}} + \frac{1}{\sqrt{7}+\sqrt{5}} + \frac{1}{\sqrt{5}+\sqrt{3}} + \frac{1}{\sqrt{3}+1}$. 10

(ii) If in a code language, COULD is written as BNTKC and MARGIN is written as LZQFHM, how will MOULDING be written in that code?

(iii) A discount of 20% is given on the marked price of an article. The shopkeeper charges sales tax of 10% on the discounted price. If the selling price be ₹1848, what is the marked price (in rupees) of the article? (CO5)

8-b. (i) A person multiplied a number by 3/5 instead of 5/3, What is the percentage error in the calculation? 10

(ii) Find the missing value 2, 9, 23, 3, 8, 25, 4,_____.

(iii) All the codes given below are only in two letters format. "Banks are digital today" is written as "Zi Li Ki Ti", "Money transfer through banks" is written as "Di Ki Si Fi", "Digital money easy today" is written as "Si Zi Ti Bi" and "Today we have leave" is written as "Gi Xi Vi Zi". What is the possible code for "Easy for transfer"? (CO5)