NOIDA INSTITUTE OF ENGINEERING AND TECHNOLOGY, GREATER NOIDA
(An Autonomous Institute)
Affiliated to Dr. A.P.J. Abdul Kalam Technical University, Uttar Pradesh, Lucknow
M.Tech(Integrated)
SEM: I - THEORY EXAMINATION (2021 - 2022)
Subject: Engineering Mathematics - I

Time: 03:00 Hours   Max. Marks: 100

General Instructions:

1. All questions are compulsory. It comprises of three Sections A, B and C.

- Section A - Question No- 1 is objective type question carrying 1 mark each & Question No- 2 is very short type questions carrying 2 marks each.
- Section B - Question No- 3 is Long answer type - I questions carrying 6 marks each.
- Section C - Question No- 4 to 8 are Long answer type - II questions carrying 10 marks each.
- No sheet should be left blank. Any written material after a Blank sheet will not be evaluated/checked.

SECTION A   20

1. Attempt all parts:-

1-a.   1

The rank of matrix $\begin{bmatrix} 5 & 10 \\ 3 & 6 \end{bmatrix}$ is (CO1)

    1. 2
    2. 3
    3. 0
    4. 1

1-b.   If the eigen values of a matrix A are 4, 5, 7 then write the eigen values of A^{-1} are (CO1)   1
    1. $4, 5^2, 7^3$
    2. 4, 5, 7
    3. 1/4, 1/5, 1/7
    4. none of these

1-c.   1

If $u = \sin^{-1}\frac{x}{y} + \tan^{-1}\frac{y}{x}$, then the value of $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}$ is (CO2)

    1. 0
    2. u
    3. 4
    4. 1

1-d.   Asymptotes parallel to y-axis of the curve $y^2(a+x) = x^2(3a-x)$ (CO2)   1
    1. $x = a$
    2. $x = -a$
    3. $x = 3a$
    4. $x = -3a$

1-e.   In a sphere of radius r, if r is measured as 6.5 inches with a possible error of 0.1 inch, then   1
possible error in volume is (CO3)
    1. 16.95π
    2. $16.98\ \pi$
    3. $16.9\ \pi$

4. 16.09π

1-f. If functions u, v, w of three independent variables x, y, z are not independent then the Jacobian of u, v, w with respect to x, y, z is (CO3)　　1

    1. 1

    2. 0

    3. 1/2

    4. None of these

1-g. The value of integral $\int_0^\pi \sin^3\theta \, d\theta$ is (CO4)　　1

    1. 1/2

    2. 5/3

    3. 1/3

    4. 4/3

1-h. The value of the triple integral $\int_0^1 \int_1^2 \int_2^3 xyz \, dx \, dy \, dz$ is (CO4)　　1

    1. 12/7

    2. 15/8

    3. 1

    4. 0

1-i. If 'green' is called 'white', 'white' is called 'yellow', 'yellow' is called 'blue', 'blue' is called 'pink' and 'pink' is called 'black', then what is the colour of milk ? (CO5)　　1

    1. green

    2. blue

    3. yellow

    4. pink

1-j. A man had 7 children. When their average age was 12 years, a child aged 6 years died. The average age of remaining six children is (CO5)　　1

    1. 13 years

    2. 10 years

    3. 11 years

    4. 14 years

2. Attempt all parts:-

2-a. Solve the system of equations $x + y + z = 0, x + 2y - z = 0, 2x + y + 3z = 0.$ (CO1)　　2

2-b. Find the nth differential coefficients of cos3x . cos5x. (CO2)　　2

2-c. Expand the function $f(x,y) = x^2 + xy - y^2$ by Taylor's theorem in powers of (x-1) and (y+2) upto second degree term. (CO3)　　2

2-d. Calculate the volume of the solid bounded by x = 0, y = 0, z = 0, x + y + z = 1.(CO4)　　2

2-e. Find the missing terms of 7, 12, 20, 37, 81, 206,_ ?. (CO5)　　2

SECTION B　　30

3. Answer any <u>five</u> of the following:-

3-a. Show that the equations $x - 4y + 7z = 14, 3x + 8y - 2z = 13, 7x - 8y + 26z = 5$ are not consistent.(CO1)　　6

3-b. Find the rank of matrix by reducing it to normal form $\begin{bmatrix} 1 & 2 & 1 & 0 \\ -2 & 4 & 3 & 0 \\ 1 & 1 & 2 & -8 \end{bmatrix}$. (CO1)　　6

3-c. Trace the following curve $y^2 (a - x) = x^2 (a + x)$. (CO2)　　6

3-d. If $u = \tan^{-1}\left(\dfrac{x^3 + y^3}{x - y}\right)$, prove that (CO2)　　　　6

(i) $x\dfrac{\partial u}{\partial x} + y\dfrac{\partial u}{\partial y} = \sin 2u$

(ii) $x^2\dfrac{\partial^2 u}{\partial x^2} + 2xy\dfrac{\partial^2 u}{\partial x \partial y} + y^2\dfrac{\partial^2 u}{\partial y^2} = 2\cos 3u \sin u$.

3-e. The diameter and altitude of a can in the shape of a right circular cylinder are measured as 4cm & 6cm respectively. The possible error in each measurement is 0.1 cm. Find approximately the maximum possible error in the value of volume. (CO3)　　　　6

3-f. Evaluate by changing the order of integration $\displaystyle\int_0^1 \int_{e^x}^{e^2} \dfrac{dy\, dx}{\log y}$. (CO4)　　　　6

3-g. In certain code language 'si po re' means 'book is thick', 'ti na re' means 'bag is heavy', 'ka si' means 'interesting book' and 'de ti' means 'that bag'. What should stand for 'that is interesting' in that code language? (CO5)　　　　6

<div align="center">SECTION C　　　　　　　　　　　　50</div>

4. Answer any <u>one</u> of the following:-

4-a. Verify Caley-Hamilton theorem for the matrix $A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$ and hence compute A^{-1}. Also evaluate $A^6 - 6A^5 + 9A^4 - 2A^3 - 12A^2 + 23A - 9I$. (CO1).　　　　10

4-b. Find the eigen values and eigen vectors of a matrix $\begin{bmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{bmatrix}$. (CO1)　　　　10

5. Answer any <u>one</u> of the following:-

5-a. If $y = \sin(a\sin^{-1}x)$, then find $y_n(0)$. (CO2)　　　　10

5-b. If $u = f(r)$, where $r = \sqrt{x^2 + y^2}$, prove that　　　　10

$\dfrac{\partial^2 u}{\partial x^2} + \dfrac{\partial^2 u}{\partial y^2} = f''(r) + \dfrac{1}{r}f'(r)$. (CO2)

6. Answer any <u>one</u> of the following:-

6-a. If u, v, w are the roots of the cubic equation $(\lambda - x)^3 + (\lambda - y)^3 + (\lambda - z)^3 = 0$ in λ then find $\dfrac{\partial(u, v, w)}{\partial(x, y, z)}$. (CO3)　　　　10

6-b. A rectangular box closed at the top is of given volume, what must be the dimensions so that the surface area is minimum. (CO3)　　　　10

7. Answer any <u>one</u> of the following:-

7-a. Show that $\beta(m, n) = \lceil m \lceil n / \lceil (m+n)$. (CO4)　　　　10

7-b. Apply Dirichlet's integral to find the volume and mass contained in the first octant solid region of the ellipsoid $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} + \dfrac{z^2}{c^2} = 1$ if the density at any point is $\rho(x,y,z) = kxyz$. (CO3)　　　　10

8. Answer any <u>one</u> of the following:-

8-a. (i) Pankaj purchased an item for Rs. 7500 and sold it at the gain of 24%. From that amount he purchased another item and sold it at the loss of 20%. What is his overall gain/loss?
(ii) The average of runs of a cricket player of 20 innings was 32. How many runs must he make in his next innings so as to increase his average of runs by 4 ?　　　　10

(iii) In certain code language, ROCK=47 and LATE=38. Find the code for FOOL. (CO5)

8-b.   (i) In certain code language SERIES is coded as 5625 and PIPE is coded as 2116. How will   10
WAP be coded in the same code language?

(ii) The average marks obtained by 22 candidates in an examination are 45. The average marks of the first 10 candidates are 55 and those of the last eleven are 40. The number of marks obtained by the eleventh candidate is ?

(iii) A candidate scores 25 % marks and fails by 30 marks, while another candidate who scores 50 % marks get 20 marks more than the minimum marks required to pass the examinations. Find the maximum marks for the examination. (CO5)