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NOIDA INSTITUTE OF ENGINEERING AND TECHNOLOGY, GREATER NOIDA

(An Autonomous Institute Affiliated to AKTU, Lucknow)

MCA

SEM: I - CARRY OVER THEORY EXAMINATION - AUGUST 2022

Subject: Discrete Mathematics

Time: 3 Hours

Max. Marks: 100

General Instructions:

1. The question paper comprises three sections, A, B, and C. You are expected to answer them as directed.
2. Section A - Question No- 1 is 1 marker & Question No- 2 carries 2 marks each.
3. Section B - Question No-3 is based on external choice carrying 6 marks each.
4. Section C - Questions No. 4-8 are within unit choice questions carrying 10 marks each.
5. No sheet should be left blank. Any written material after a blank sheet will not be evaluated/checked.

SECTION A

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1. Attempt all parts:-

- 1-a. Power set of empty set has exactly \_\_\_\_\_ subset. (CO1) 1
- (a) One
- (b) Two
- (c) Zero
- (d) Three
- 1-b. What type of a relation is  $R = \{(1, 3), (4, 2), (2, 4), (2, 3), (3, 1)\}$  on the set  $A = \{1, 2, 3, 4\}$ ? 1
- (CO1)
- (a) Reflexive
- (b) Transitive
- (c) Symmetric
- (d) None of these
- 1-c. What is the number of edges present in a complete graph having  $n$  vertices? (CO2) 1
- (a)  $(n*(n+1))/2$
- (b)  $(n*(n-1))/2$
- (c)  $n$

(d) Information given is insufficient

- 1-d. Total number of degrees of an isolated node is (CO2) 1
- (a) 0
  - (b) 1
  - (c) 2
  - (d) 3
- 1-e. An algebraic structure is called a semigroup. (CO3) 1
- (a)  $(P, *)$
  - (b)  $(Q, +, *)$
  - (c)  $(P, +)$
  - (d)  $(+, *)$
- 1-f. A function  $f(x)$  is defined from A to B then  $f^{-1}$  is defined \_\_\_\_\_ (CO3) 1
- (a) from A to B
  - (b) from B to A
  - (c) depends on the inverse of function
  - (d) none of the mentioned
- 1-g. A compound proposition that is neither a tautology nor a contradiction is called a (CO4) 1
- (a) Contingency
  - (b) Equivalence
  - (c) Condition
  - (d) Inference
- 1-h. Which of the proposition is  $p \wedge (\sim p \vee q)$  (CO4) 1
- (a) tautology
  - (b) A contradiction
  - (c) Logically equivalent to  $p \wedge q$
  - (d) All of the above
- 1-i. Determine the value of  $a_2$  for the recurrence relation  $a_n = 4a_{n-1} + 3$  with  $a_0 = 3$  (CO5) 1
- (a) 66
  - (b) 65
  - (c) 64
  - (d) 63

- 1-j. In how many ways can be letters of the word LEADER be arranged? (CO5) 1
- (a) 72
- (b) 144
- (c) 360
- (d) 720

2. Attempt all parts:-

- 2.a. Write an example of finite and infinite set in set builder form. (CO1) 2
- 2.b. Define Least Upper Bound with an example. (CO2) 2
- 2.c. State any two properties of a group. (CO3) 2
- 2.d. Define Tautology with an example. (CO4) 2
- 2.e. In how many ways can the letters be arranged so that all the vowels come together? Word is "HOCKEY." (CO5) 2

### SECTION B

30

3. Answer any five of the following:-

- 3 If X, Y and Z are Three sets then Prove that  $X - (Y \cap Z) = (X - Y) \cup (X - Z)$  (CO1) 6
- 3 R and S are relation on  $A = \{ 1, 2, 3 \}$ ,  $R = \{ (1,1), (1,2), (2,3), (3,1), (3,3) \}$  and  $S = \{ (1, 2), (1, 3), (2, 1), (3, 3) \}$  then find  $R \circ S$  and  $S \circ R$  (CO1) 6
- 3 Draw the binary search tree for the following input list 60, 25,75,15,50,66,33,44. Trace an algorithm to delete the nodes 25, 75, 44 from the tree. (CO2) 6
- 3 Draw the Hasse diagram of the poset  $(S, \leq)$  where  $S = \{1, 3, 5, 7, 9, 14, 18\}$  and  $X \leq Y$  if X divides Y. (CO2) 6
- 3.e. Prove that intersection of two normal subgroups of a group G is a normal subgroup of G. (CO3) 6
- 3.f. Prove that  $(A \vee B) \wedge [(\neg A) \wedge (\neg B)]$  is a contradiction (CO4) 6
- 3.g. Solve the recurrence relation  $2a_r - 5a_{r-1} + 2a_{r-2} = 0$  then find the particular solution  $a_0 = 0$  and  $a_1 = 1$ . (CO5) 6

### SECTION C

50

4. Answer any one of the following:-

- 4 Prove using mathematical induction that for all  $n \geq 1$ ,  $1 + 4 + 7 + \dots + (3n - 2) = n(3n - 1)/2$ . (CO1) 10
- 4 Show that the relation "is proper subset of" with respects to sets is not an equivalence 10

relation. (CO1)

5. Answer any one of the following:-

5 Show that a complemented distributive lattice is a boolean algebra. (CO2) 10

5 Prove that any connected graph G with N vertices and N -1 Edges in a tree. (CO2) 10

6. Answer any one of the following:-

6 Show that the set  $G = \{1, 2, 3, 4\}$  is a finite abelian group under multiplication modulo 5. (CO3) 10

6 Define order of an element of a group, find the order of every element in the multiplicative group  $G = \{ a , a^2 , a^3 , a^4 , a^5 , a^6 = e \}$ . (CO3) 10

7. Answer any one of the following:-

7 Use the truth tables method to determine whether  $(\neg p \vee q) \wedge (q \rightarrow \neg r \wedge \neg p) \wedge (p \vee r)$  (denoted with  $\phi$ ) is satisfiable. (CO4) 10

7 Are the statements  $P \rightarrow (Q \vee R)$  and  $(P \rightarrow Q) \vee (P \rightarrow R)$  logically equivalent or not ? (CO4) 10

8. Answer any one of the following:-

8 How many 3-digit numbers can be formed from the digits 2, 3, 5, 6, 7 and 9, which are divisible by 5 and none of the digits is repeated? (CO5) 10

8 Solve the recurrence relation  $U^{r+2} - U^{r+1} + 6U^r = 2^r + r$  (CO5) 10