Printed Page:- Subject Code:- AAS0204

Roll. No:

NOIDA INSTITUTE OF ENGINEERING AND TECHNOLOGY, GREATER NOIDA

(An Autonomous Institute Affiliated to AKTU, Lucknow)

B.Tech

SEM: II - CARRY OVER THEORY EXAMINATION - SEPTEMBER 2022

Subject: Mathematical Foundations – II

Time: 3 Hours Max. Marks: 100

General Instructions:

1. The question paper comprises three sections, A, B, and C. You are expected to answer them as directed.
2. Section A - Question No- 1 is 1 marker & Question No- 2 carries 2 mark each.
3. Section B - Question No-3 is based on external choice carrying 6 marks each.
4. Section C - Questions No. 4-8 are within unit choice questions carrying 10 marks each.
5. No sheet should be left blank. Any written material after a blank sheet will not be evaluated/checked.

SECTION A 20

1. Attempt all parts:-

1-a. The value of integral $\int_1^0 \int_0^1 (x+y)\, dx\, dy$ is (CO1) 1

(a) 2

(b) -1

(c) -2

(d) None of these

1-b. The value of integral $\int\int\int dx\, dy\, dz$ *where* R: $-1 \le x \le 1, -2 \le y \le 2, -3 \le z \le 3$ is (CO1) 1

(a) 24

(b) 48

(c) -24

(d) -48

1-c. The solution of the differential equation $(D^5 - D^3)y = 0$ will be (CO2) 1

(a) $y = (c_1 + c_2 x + c_3 x^2)e^x + c_4 e^x + c_5 e^{-x}$

(b) $y = (c_1 + c_2 x + c_3 x^2)e^{-x} + c_4 e^x + c_5 e^{-x}$

(c) $y = c_1 + c_2x + c_3x^2 + c_4e^x + c_5e^{-x}$

(d) None of these

1-d. Part of the C.F. of $xy'' - y' + (1-x)y = x^2e^{-x}$ by the method of reduction of order is (CO 2) 1

(a) e^{-x}

(b) e^x

(c) x

(d) x^2

1-e. 1

The Order of the partial differential equation $\left(\frac{\partial^2 z}{\partial x^2}\right)^2 + \frac{\partial^2 z}{\partial x \partial y} - \frac{\partial z}{\partial x} = e^{x+y}$ is (CO3)

(a) 1

(b) 2

(c) 3

(d) None of these

1-f. Which of the following is the correct partial differential equation of the relation z = (x + a)(y + b), where 'a' and 'b' are constant (CO3) 1

(a) $z = pq$

(b) $z = p + q + ab$

(c) $z = px + qy + pq$

(d) $z = px + qy - pq$

1-g. 1

Inverse Laplace of the function $f(s) = \left[\frac{1}{s(s^2+1)}\right]$ is (CO 4)

(a) $1 - \cos t$

(b) $1 + \sin t$

(c) $1 - \sin t$

(d) None of these

1-h. Laplace transform of $e^{-3t}u(t-2)$ is (CO 4) 1

(a) $\frac{e^{-2(s+3)}}{s-3}$

(b) $\frac{e^{-2(s+3)}}{s+3}$

(c) $\frac{e^{-2(s+3)}}{s^2+3}$

(d) $\frac{e^{-2(s+3)}}{s^2-3}$

1-i. The ratio of present ages of Sri and Gowtham is 3: 4. Mahesh is 6 years older than Sri and two years younger than Gowtham. The sum of the present ages of Sri and Mahesh is (CO5) 1

(a) 48 years

(b) 50 years

(c) 52 years

(d) 54 years

1-j. A watch reads 4:30. If the minute hand point East, in what direction will the hour hand point? (CO5) 1

(a) South-East

(b) North-East

(c) South- West

(d) North-West

2. Attempt all parts:-

2.a. Evaluate the integral. $\int_0^1 \int_0^1 \frac{dx\ dy}{\sqrt{1-x^2}\sqrt{1-y^2}}$. (CO1) 2

2.b. Find the particular integral of differential equation $(D^2-6D+9)y = exp(3x)$. (CO2) 2

2.c. Form the partial differential equation by eliminating the arbitrary function from the equation 2

$z = f(x^2-y^2)$ (CO3)

2.d. Find Laplace transform of the function F(t) = cosh at cos at. (CO 4) 2

2.e. Gopal starts from his house towards West. After walking a distance of 30 m, he turned towards right and walked 20 m. He then turned left and moving a distance of 10 m, turned to his left again and walked 40 m. He now turns to the left and walks 5 m. Finally he turns to his left. In which direction is he walking now? (CO5) 2

SECTION B 30

3. Answer any <u>five</u> of the following:-

3-a. Evaluate $\int_0^\infty \frac{x^8(1-x^6)}{(1+x)^{24}}\,dx$. (CO1) 6

3-b. The plane $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$ meets the coordinate axes in A, B, C. Find the volume of tetrahedron OABC. (CO1) 6

3-c. Solve $(D^2-3D+2)y = x^2+2x+1$. (CO2) 6

3-d. Solve the differential equation $x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}+y=\sin(\log x^2)$. (CO2) 6

3.e. Solve the linear partial differential equation $(D-3D'-2)^2z=2e^{2x}\tan(y+3x)$. (CO3) 6

3.f. Find inverse Laplace Transform of the function $f(s)=\log\frac{s(s+1)}{s^2+4}$. (CO 4) 6

3.g. (i) If the compound interest on a certain sum for 2 years at 3% per annum is 101.50, then find the simple interest on the same sum at the same rate and for the same time? 6

(ii) On a certain sum of money the compound interest for 2 years is Rs. 282.15 and the simple interest for the same period of time is Rs. 270. Find the rate of interest per annum? (CO5)

SECTION C 50

4. Answer any <u>one</u> of the following:-

4-a. Change the order of integration for $I=\int_0^1\int_{x^2}^{2-x}xy\,dy\,dx$ and evaluate the same. (CO1) 10

4-b. Apply Dirichlet's integral to evaluate $\iiint x^2yz\,dx\,dy\,dz$, throughout the volume bounded by the planes $x=0, y=0, z=0$ and $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$. (CO1) 10

5. Answer any <u>one</u> of the following:-

5-a. Solve the differential equations by method of variation of parameters $y''+y=\sec x$ (CO2) 10

5-b. Solve the following differential equation by changing the independent variable $\frac{d^2y}{dx^2}-\frac{1}{x}\frac{dy}{dx}+4x^2y=x^4$. (CO 2) 10

6. Answer any <u>one</u> of the following:-

6-a. Solve the linear partial differential equation $\frac{\partial^2z}{\partial x^2}+\frac{\partial^2z}{\partial x\partial y}-2\frac{\partial^2z}{\partial y^2}=(y-1)e^x$. (CO3) 10

6-b. Solve : $(D^2-D'^2-3D+3D')z=xy+e^{x+2y}$. (CO3) 10

7. Answer any <u>one</u> of the following:-

7-a. Solve the following simultaneous differential equation by using Laplace transform $\frac{dx}{dt}-y=e^t$ & $\frac{dy}{dt}+x=\sin t$, *Given that* $x(0)=1, y(0)=0$. (CO4) 10

7-b. State convolution theorem and hence evaluate $L^{-1}\left\{\frac{s}{(s^2+1)(s^2+4)}\right\}$. (CO4) 10

8. Answer any one of the following:-

8-a. (i) Prerna invested Rs x for 6 months, Ankita Rs 2400 for 10 months and Pavneet Rs 3900 for 8 months. If Ankita got Rs 6000 out of a total profit of Rs 19,200, then what is the money? 10

(ii) Trisha and Misha invested Rs 3500 and Rs 3000 in a business. After 7 months both added Rs 500 to their investments. If after a year the difference in their shares of profit is Rs 1140,find the total profit at the end of year? (CO5)

8-b. (i) Two vessel contain milk and water in ratio 3:2 and 7:3. Find the ratio in which the contents of the two vessels have to be mixed to get a new mixture in which the ratio of milk and water is 2:1. 10

(ii) Alloy A contains 40% gold and 60% silver. Alloy B contains 35% gold and 40% silver and 25% copper. Alloy A and B are mixed in the ratio of 1:4. What is the ratio of gold and silver in the newly formed alloy? (CO5)