NOIDA INSTITUTE OF ENGINEERING AND TECHNOLOGY, GREATER NOIDA

(An Autonomous Institute Affiliated to AKTU, Lucknow)

B.Tech

SEM: II - THEORY EXAMINATION (2021 - 2022)

Subject: Linear Algebra

Time: 3 Hours

Max. Marks: 100

General Instructions:

1. The question paper comprises three sections, A, B, and C. You are expected to answer them as directed.

2. Section A - Question No- 1 is 1 marker & Question No- 2 carries 2 marks each.

3. Section B - Question No-3 is based on external choice carrying 6 marks each.

4. Section C - Questions No. 4-8 are within unit choice questions carrying 10 marks each.

5. No sheet should be left blank. Any written material after a blank sheet will not be evaluated/checked.

SECTION A 20

1. Attempt all parts:-

1   The matrix $A = \begin{bmatrix} p & -q \\ q & p \end{bmatrix}$ is orthogonal if and only if (CO1)   1

    (a) $p^2 + q^2 = 1$

    (b) $p^2 = q^2$

    (c) $p^2 = q^2 + 1$

    (d) none of these

1   The matrix $A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$ is (CO1)   1

    (a) Nilpotent matrix

    (b) Idempotent matrix

    (c) Involuntary matrix

    (d) None of these

1-c.   The rank of matrix $A = \begin{bmatrix} 5 & 10 \\ 3 & 6 \end{bmatrix}$ is (CO2)   1

    (a) 6

    (b) 5

(c) 1

(d) none of these

1-d. The System of equations x+2y+3z=1,2x+y+3z=2,5x+5y+9z=4 has (CO2)   1

(a) only one solution

(b) infinitely many solutions

(c) no solution

(d) none of these

1-e. Which of the set of vectors are linearly dependent? (CO3)   1

(a) (1, 1, -1), 2, -3, 5), (-2, 1, 4)

(b) (1, -1, -1), (2, -3, 5), (-2, 1, 4)

(c) (1, 4, -1), 2, -2, 5), (-2, 1, 4)

(d) None of these

1-f. A subset S of a vector space V is said to be basis if (CO3)   1

(a) S is L.I. and L(S) = 0

(b) S is L.I. and L(S) = S

(c) S is L.D. and L(S) = V

(d) S is L.I. and L(S) = V

1-g. The eigen values of $4A^{-1} + 3A + 2I$, where $A = \begin{bmatrix} 1 & 0 \\ 2 & 4 \end{bmatrix}$ are (CO4)   1

(a) 1, 2

(b) 9, 15

(c) 3, 4

(d) None of these

1-h. Let V and W be vector space over T: V→W be a map. Then T is a linear transformation iff (CO4)   1

(a) $T(ax) = T(x)$

(b) $T(a + \beta) = T(a) + T(\beta)$

(c) $T(aa + b\beta) = a\,T(a) + bT(\beta)$

(d) None of these

1-i. If $A = \begin{bmatrix} 3 & 0 \\ 4 & 5 \end{bmatrix}$ then the Eigen value of $A^{T}A$ are (CO5)   1

(a) 45 ,5

(b) $45, 45$

(c) $5, 5$

(d) None of these

1-j.   In singular value decomposition method USV^T, where S is (CO5)   1

(a) Orthogonal

(b) Transpose of orthogonal matrix

(c) Diagonal matrix

(d) None of these

2. Attempt all parts:-

2.a.	Explain symmetric matrix and orthogonal matrix with example.(CO1)	2
2.b.	Show that the system of vectors X_1=(1,2,3) and (4,-2,7) are linearly independent.(CO2)	2
2.c.	Define Vector space.(CO3)	2
2.d.	Explain Skew Hermitian matrix with example. (CO4)	2
2.e.	Find the singular values of matrix $A = \begin{bmatrix} 1 & 3 \\ 3 & 1 \end{bmatrix}$.(CO5)	2

<div align="center">SECTION B   30</div>

3. Answer any <u>five</u> of the following:-

3-a.	Find the inverse of the matrix $\begin{bmatrix} 2 & 3 & 4 \\ 4 & 3 & 1 \\ 1 & 2 & 4 \end{bmatrix}$ by elementary transformation.(CO1)	6
3-b.	Solve the following equations by Cramer's rule- (CO1) x+y+z=6, 2x+3y-z=5 and 6x-2y-3z=-7	6
3-c.	**Find the LU decomposition of a matrix** $\begin{bmatrix} 1 & -1 \\ 2 & -1 \end{bmatrix}$.(CO2)	6
3-d.	**Find the values of a and b such that the rank of matrix** $\begin{bmatrix} 1 & -2 & 3 & 1 \\ 2 & 1 & -1 & 2 \\ 6 & -2 & a & b \end{bmatrix}$ is 2.(CO2)	6
3.e.	Show that the vectors (2,1,4), (1,-1,2) and (3,1,-2) forms a basis of R^3.(CO3)	6
3.f.	Check whether the following transformation is linear or not $T:V_2(R) \rightarrow V_2(R)$ defined as $T(x,y) = (x^2, y^2)$. (CO4)	6
3.g.	**Given the following data, Using PCA find the covariance.**(CO5) x: 4 8 13 7 y: 11 4 5 14	6

<div align="center">SECTION C   50</div>

4. Answer any <u>one</u> of the following:-

4-a.   Solve the system of equations by matrix method: (CO1)   10

x+2y-3z=4, 2x+3y+2z=2 and 3x-3y-4z=11.

4-b.   Find the inverse of the matrix A by applying elementary transformations. (CO1)   10

$$A = \begin{bmatrix} 2 & 1 & -1 & 2 \\ 1 & 3 & 2 & -3 \\ -1 & 2 & 1 & -1 \\ 2 & -3 & -1 & 4 \end{bmatrix}.$$

5. Answer any <u>one</u> of the following:-

5-a.   Find the rank of a matrix reducing to normal form $A = \begin{bmatrix} 2 & 1 & -3 & -6 \\ 3 & -3 & 1 & 2 \\ 1 & 1 & 1 & 2 \end{bmatrix}$.(CO2)   10

5-b.   Determine the value of λ and μ so that the equations x + y + z = 6, x + 2y + 3z = 10, x + 2y   10

+ λz = μ. (CO2)

(i) No solution

(ii) Unique solution

(iii) Infinite solution

6. Answer any <u>one</u> of the following:-

6-a.   If W_1 and W_2 are subspaces of the vector space $R^4(R)$ generated by   10

S_1={(1,1,0,-1), (1,2,3,0), (2,3,3,-1)}, S_2={(1,2,2,-2), (2,3,2,-3), (1,3,4,-3)} respectively,

Determine-(CO3)

(a) dim($W_1 + W_2$)

(b) dim($W_1 \cap W_2$)

6-b.   Apply Gram-schmidt process to the vectors α_1=(1,0,1), α_2=(1,0,-1), α_3=(0,3,4) to obtain   10

the orthonormal basis for V_3(R).(CO3)

7. Answer any <u>one</u> of the following:-

7-a.   Find the eigen values and eigen vector of the matrix $A = \begin{bmatrix} 3 & 0 & 0 \\ 1 & -2 & -8 \\ 0 & -5 & 1 \end{bmatrix}$. (CO4)   10

7-b.   Is the following matrix positive definite? (CO4)   10

$$\begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}$$

8. Answer any <u>one</u> of the following:-

8-a.   Find a singular value decomposition of $A = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$.(CO5)   10

8-b. Given the following data, use PCA to reduce the dimension from 2 to 1.(CO5)   10

Feature	Example 1	Example 2	Example 3	Example 4
x:	4	8	13	7
y:	11	4	5	14