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NOIDA INSTITUTE OF ENGINEERING AND TECHNOLOGY, GREATER NOIDA

(An Autonomous Institute Affiliated to AKTU, Lucknow)

M.Tech (Integrated)

SEM: II - THEORY EXAMINATION (2021 - 2022)

Subject: Engineering Mathematics-II

Time: 3 Hours Max. Marks: 100

General Instructions:

1. The question paper comprises three sections, A, B, and C. You are expected to answer them as directed.
2. Section A - Question No- 1 is 1 marker & Question No- 2 carries 2 marks each.
3. Section B - Question No-3 is based on external choice carrying 6 marks each.
4. Section C - Questions No. 4-8 are within unit choice questions carrying 10 marks each.
5. No sheet should be left blank. Any written material after a blank sheet will not be evaluated/checked.

SECTION A 20

1. Attempt all parts:-

1-a. Degree and order of the differential equation $\sqrt{((dy/dx)^2+3y)} = (d^2y)/(dx^2)$ is (CO1) 1

(a) Ord = 2, Deg = 2

(b) Ord = 2, Deg = 1

(c) Ord = 1, Deg = 2

(d) Ord = 1, Deg = 1

1-b. The P.I. of the differential equation $(D^2 + 6D + 9)y = 5e^{3x}$ (CO1) 1

(a) $(5/36)e^{3x}$

(b) $(1/36)e^{3x}$

(c) $(5x/36)e^{3x}$

(d) None of these

1-c. The Series $1+\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{3}}+\frac{1}{\sqrt{4}}+\frac{1}{\sqrt{5}}+\ldots\ldots$ is (CO2) 1

(a) convergent

(b) oscillatory

(c) divergent

(d) none of these

1-d. The coefficient a_0 in a Fourier series for the function $f(x) = x + x^3$ in the interval $-\pi < x < \pi$ is (CO2) 1

(a) π

(b) 2π

(c) 0

(d) none of these

1-e. Laplace transform of te^{-t} is (CO3) 1

(a) $\frac{1}{(s+1)^2}$

(b) $\frac{2}{(s+1)^2}$

(c) $\frac{1}{(s-1)^2}$

(d) None of these

1-f. Laplace transform of $e^{-3t}u(t-2)$ is (CO3) 1

(a) $\frac{e^{-2(s+3)}}{s-3}$

(b) $\frac{e^{-2(s+3)}}{s+3}$

(c) $\frac{e^{-2(s+3)}}{s^2+3}$

(d) $\frac{e^{-2(s+3)}}{s^2-3}$

1-g. The directional derivative of $\frac{1}{r}$ in the direction of $\vec{r}$ where $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ is (CO4) 1

(a) $-\frac{1}{r^2}$

(b) $-\frac{5}{r^4}\vec{r}$

(c) $-\frac{2}{r^5}\vec{r}$

(d) $\frac{8}{r^4}\vec{r}$

1-h. Divergence of $2x^2z\hat{i} - xy^2z\,\hat{j} + 3yz^2\hat{k}$ at the point $(1,1,1)$ is (CO4) 1

(a) 0

(b) 12

(c) 3

(d) 8

8

1-i. A person starts towards South direction. Which of the following order of direction will lead him to East direction? (CO5) 1

(a) Right, Right, Right

(b) Left, Left, Left

(c) Left, Right, Right

(d) Right, Left, Right

1-j. Find the simple interest on Rs 500 for 5 years at 10% per annum. (CO5) 1

(a) Rs 500

(b) Rs 125

(c) Rs 250

(d) Rs 350

2. Attempt all parts:-

2.a. Solve the differential equation: $(D^2-2D+4)^2 y=0.$ (CO1) 2

2.b. Test the convergence of the series $\sum_{n=1}^{\infty} u_n$ where $u_n=\frac{1+(n+1)}{1+(n+1)^2}$. (CO2) 2

2.c. Find the inverse Laplace transform of the function $f(s)=\frac{1}{s^{3/2}}+\frac{1}{s-4}$. (CO3) 2

2.d. Show that vector $\vec{V}=(x+3y)\hat{i}+(y-3z)\hat{j}+(x-2z)\hat{k}$, is solenoidal. (CO4) 2

2.e. If P $ Q means P is the father of Q; P # Q means P is the mother of Q and P * Q means P is the sister of Q, then N # L $ P * Q shows which of the relation of Q to N? (CO5) 2

SECTION B 30

3. Answer any <u>five</u> of the following:-

3-a. Solve the differential equation $x^2\frac{d^2y}{dx^2}+4x\frac{dy}{dx}+2y=e^x$ (CO1) 6

3-b. Solve $\frac{dx}{dt}-y=e^t,\ \frac{dy}{dt}-x=\sin t;\ x(0)=1, y(0)=0.$ (CO1) 6

3-c. Test the convergence of the series $\sum_{n=1}^{\infty}\frac{(n+1)}{n^3}x^n$ (CO2) 6

3-d. Expand f(x) = π x - x^2 as a Fourier half range sine series in 0 < x < π upto the first three terms. (CO2) 6

3.e. Find the Laplace Transform of the function $F(t)=\int_0^t te^{-t}\sin 4t\,dt$. (CO3) 6

6

3.f Find the total work done by a force $\vec{F} = (x^2 + y^2)\hat{i} - 2xy\hat{j}$ in moving a point from (0,0) to (a,b) along the rectangle bounded by the lines x=0, x=a, y=0, and y=b. (CO4)

3.g. (i) Lalit along with his family decided to take a road trip to a nearby resort and spend the weekend there. He started from his home and from there drove 70 km to the south, he then took a right turn and drove 30 km. Next, he took a right turn and drove 30 km and stopped at a restaurant. What is the shortest distance between his house and the restaurant? (CO5) 6

(ii) Two customers borrowed the same amount of money, one at compound interest and at the other at simple interest. If after 2 years, the interest payable by one was Rs. 220 and the other Rs. 200, then what was the principle money lent to each of them? (CO5)

SECTION C 50

4. Answer any one of the following:-

4-a. Solve the differential equation in series $xy'' + y' - y = 0$ about $x = 0$. (CO1) 10

4-b. Solve the differential equations by method of variation of parameters $y'' - y = \frac{2}{1+e^x}$. (CO1) 10

5. Answer any one of the following:-

5-a. Test the convergence the series, 10

$$1 + \frac{2}{3}.\frac{1}{4} + \frac{2.4}{3.5}.\frac{1}{6} + \frac{2.4.6}{3.5.7}.\frac{1}{8} + \frac{2.4.6.8}{3.5.7.9}.\frac{1}{10} + \ldots\ldots$$ (CO2)

5-b. Obtain the Fourier series to represent function f(x) = x sinx in the interval $-\pi \le x \le \pi$. 10

Hence show that

$$\frac{1}{1.3} - \frac{1}{3.5} + \frac{1}{5.7} - \frac{1}{7.9} + \ldots\ldots = \frac{\pi - 2}{4}.$$ (CO2)

6. Answer any one of the following:-

6-a. Solve the following differential equation by using Laplace transform 10

$y'' + 3y' + 2y = te^{-t}$, where $y(0) = 1, y'(0) = 0$. (CO3)

6-b. By using Convolution theorem, find $L^{-1}\left\{\frac{s}{(s^2+4)(s^2+9)}\right\}$. (CO3) 10

7. Answer any one of the following:-

10

7-a. Apply Stokes theorem to evalaute $\int_C (x+y)\,dx + (2x-z)\,dy + (y+z)\,dz$, where C is the boundary of the triangle with vertices (2,0,0), (0,3,0), (0,0,6). (CO4)

7-b. Verify Divergence theorem for $\vec{F} = 4xz\,\hat{i} - y^2\,\hat{j} + yz\,\hat{k}$ taken over the cube bounded by the planes x = 0, x = 1, y = 0, y = 1, z = 0, z = 1. (CO4) 10

8. Answer any <u>one</u> of the following:-

8-a. (i) The present ages of three persons, Raj, Rajesh and Ravi is in proportions 4: 7: 9. Eight years ago, the sum of their ages was 56. What is the present age of Ravi? (CO5) 10

(ii) The ratio between two numbers is 3:4. If each number is increased by 2, the ratio becomes 7:9. Find the numbers. (CO5)

8-b. (i) Three vessels containing mixtures of milk and water are of capacities which are in the ratio 1:2:3. The ratios of milk and water in the three vessels are 4:1, 3:2 and 2:3 respectively. If one-fourth contents of first vessel, one-third of that of second vessel and half of that of third vessel are mixed; what is the ratio of milk and water in the new mixture? (CO5) 10

(ii) A, B and C enter into a partnership by investing 1500, 2500 and 3000 rupees respectively. A as manager gets one-tenth of the total profit and remaining profit is divided among the three in the ratio of their investment. If A's total share is Rs. 369, find the shares of B and C. (CO5)