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NOIDA INSTITUTE OF ENGINEERING AND TECHNOLOGY, GREATER NOIDA

(An Autonomous Institute Affiliated to AKTU, Lucknow)

B.Tech

SEM: II- THEORY EXAMINATION (2022-2023)

Subject: Engineering Mathematics - II

Time: 3 Hours   Max. Marks: 100

General Instructions:

IMP: Verify that you have received question paper with correct course, code, branch etc.

1. *This Question paper comprises of three Sections -A, B, & C. It consists of Multiple Choice Questions (MCQ's) & Subjective type questions.*
2. *Maximum marks for each question are indicated on right hand side of each question.*
3. *Illustrate your answers with neat sketches wherever necessary.*
4. *Assume suitable data if necessary.*
5. *Preferably, write the answers in sequential order.*
6. *No sheet should be left blank. Any written material after a blank sheet will not be evaluated/checked.*

SECTION – A   20

1. Attempt all parts:-

1-a.   The following differential equation has $3\frac{d^2y}{dx^2} + 3(\frac{dy}{dx})^3 + y^2 + 2 = x^5$ (CO1)   1

      (a) degree=2, order=1

      (b) degree=1, order=2

      (c) degree=3, order=1

      (d) None of these

1-b.   A solution of the following differential equation is given by   (CO1)   1

$$\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 6y = 0$$

      (a) $y = c_1e^{2x} + c_2e^{-3x}$

      (b) $y = c_1e^{2x} + c_2e^{3x}$

      (c) $y = c_1e^{-2x} + c_2e^{-3x}$

      (d) None of these

1-c.   State, which one of the alternatives is correct : The series 1-1+1-1+1…… is   (CO2)   1

      (a) Convergent with its sum 0

      (b) Convergent with its sum 1

      (c) Divergent

      (d) Oscillatory

1-d. The values of a_0 and a_n for the function $f(x) = x^3$ in the interval $-\pi < x < \pi$ are   1
(CO2)

    (a) $0, 0$

    (b) $1, 2$

    (c) $0, 3$

    (d) None of these

1-e. If $L\{F(t)\} = f(s)$, then $L\{F(at)\}$ is   (CO3)   1
(a) $\frac{1}{a} f(\frac{s}{a})$
(b) $\frac{1}{a} f(s - a)$
(c) $f(\frac{s}{a})$
(d) None of these

1-f. Laplace transform of Unit Step function $u(t - a)$ is   (CO3)   1
(a) $\frac{1}{s} e^{as}$
(b) $\frac{1}{s} e^{-as}$
(c) e^{as}
(d) None of these

1-g. If $\vec{v} = yzi + 3\,xzj + zk$, then $curl\ \vec{v}$ is   (CO4)   1
    (a) $3xi - yj + 2zk$

    (b) $-3xi + yj + 2zk$

    (c) $3xi - yj - 2zk$

    (d) None of these

1-h. A vector field with vanishing curl is called an   (CO4)   1

    (a) Rotational vector

    (b) Irrotational vector

    (c) Solenoidal vector

    (d) None of these

1-i. If $a : b = 5 : 9$ and $b : c = 7 : 4$ then $a : b : c$ is   (CO5)   1

    (a) $35 : 63 : 36$

    (b) $36 : 63 : 36$

    (c) $35 : 33 : 36$

    (d) None of these

1-j. Ravi lent Rs. 5000 to Santosh for 3 years at the rate of 5% per annum compound   1

interest. Calculate the amount that Ravi will get after 3 years.   (CO5)

a) 5788

b) 5788.125

c) 5788.135

d) None of these

2. Attempt all parts:-

2.a. The particular integral (PI) of the differential equation   (CO1)   2

$x^2 \frac{d^2y}{dx^2} - 2x \frac{dy}{dx} + 2y = 4$ is…… .

2.b. Test the convergence of the series $\frac{4}{1} + \frac{5}{4} + \frac{6}{9} + \frac{7}{16} + \cdots$ (CO2)   2

2.c. Find the Laplace transform of $\{e^{2t} t^2\}$.   (CO3)   2

2.d. Write the statement of Gauss Divergence theorem.   (CO4)   2

2.e. In what ratio must a shopkeeper mix two types of rice worth Rs. 50 kg and Rs. 70 kg, so that   2
the average cost of the mixture is Rs. 65 kg?   (CO5)

SECTION – B   30

3. Answer any <u>five</u> of the following-

3-a. Solve the differential equation $(D - 2)^2 y = e^{3x} \sin 2x$.   (CO1)   6

3-b. Solve the simultaneous differential equation $\frac{dx}{dt} + 5x - 2y = t, \frac{dy}{dt} + 2x + y = 0$.   6
(CO1)

3-c. Test the convergence of the series $\frac{1}{1.2.3} + \frac{1}{2.3.4} + \frac{1}{3.4.5} + \cdots ....$. (CO2)   6

3-d. Obtain the half range sine and cosine series for the function   (CO2)   6
$f(x) = x$ in $0 < x < 2$.

3-e. Find the Laplace transform of $\int_0^t e^t \frac{\sin t}{t} \, dt$.   (CO3)   6

3-f. Find the directional derivative of the function $\phi = x^2 - y^2 + 2z^2$ at the point   6
P(1, 2, 3) in the direction of the line PQ where Q is the point (5, 0, 4). (CO4)

3-g. The product of the ages of Aaisha and Niti is 540. If twice the age of Aaisha is more   6
than Niti's age by 6 years, then find Aaisha's age?   (CO5)

SECTION – C   50

4. Answer any <u>one</u> of the following-

4-a. Solve the differential equation $\cos x \frac{d^2y}{dx^2} + \sin x \frac{dy}{dx} - 2y \cos^3 x = 2\cos^5 x$ by   10
changing into normal form.   (CO1)

4-b. Solve the differential equation $\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = \frac{e^x}{1+e^x}$ by the method of variation   10
of parameters.   (CO1)

5. Answer any <u>one</u> of the following-

5-a. Test the convergence of the following series: 10
$1 + \frac{3}{7}x + \frac{3.6}{7.10}x^2 + \frac{3.6.9}{7.10.13}x^3 + \cdots ....; x > 0.$ (CO2)

5-b. Find the Fourier series of the periodic function f with period 2π defined as follows: 10

$$f(x) = \begin{cases} 0, & for \ -\pi < x \leq 0, \\ x, & for \ 0 \leq x \leq \pi \end{cases}$$. Hence prove that $1 + \dfrac{1}{3^2} + \dfrac{1}{5^2} + \dfrac{1}{7^2} + \dots = \dfrac{\pi^2}{8}$. (CO2)

6. Answer any <u>one</u> of the following-

6-a. Use convolution theorem to evaluate $L^{-1}\left\{\dfrac{s}{(s^2+9)^2}\right\}$.   (CO3)   10

6-b. Solve the differential equation using Laplace transform $y'' + 2y' + y = te^{-t}$, if 10
       $y(0) = 1, y'(0) = 2$.   (CO3)

7. Answer any <u>one</u> of the following-

7-a. Verify Stoke's theorem for $\vec{F} = (x^2 + y^2)i - 2\,xyj$ taken round the rectangle 10
       bounded by the lines $x = \pm a, y = 0. y = b$.   (CO4)

7-b. Apply Green's theorem to evaluate $\int[(2x^2 - y^2)dx + (x^2 + y^2)dy]$ over the region 10
       C, where C is the boundary of the area enclosed by the x-axis and the upper half of
       circle $x^2 + y^2 = a^2$.   (CO4)

8. Answer any <u>one</u> of the following-

8-a. (i) The population of a town decreased every year due to migration, poverty and 10
       unemployment. The present population of the town is 6,31,680. Last year the
       migration was 4%, and the year before last, it was 6%. What was the population two
       years ago?
       (ii) Sanjeev walks 10 m towards the South. Turning to the left, he walks 20 m and then
       moves to his right. After moving a distance of 20 m, he turns to the right and walks
       20 m. Finally, he turns to the right and moves a distance of 10 m. How far and in
       which direction is he from the starting point?   (CO5)

8-b. (i) A container contains a mixture of two liquids P and Q in the ratio of 7:5. When 10
       9 liters of mixture is taken out and replaced with Q, the ratio becomes 7:9. Find the
       quantity of liquid P in the container.

       (ii) A container contains 40 litres of milk. From this container, 4 litres of milk was
       taken out and replaced by water. This process was repeated further two times. How
       much milk is now contained by die container?   (CO5)