Printed page: 05 Subject Code- AAS0204

Roll No:

NOIDA INSTITUTE OF ENGINEERING AND TECHNOLOGY, GREATER NOIDA

(An Autonomous Institute Affiliated to AKTU, Lucknow)

B.Tech.

SEM: I - THEORY EXAMINATION (2022-2023)

Subject : Mathematical foundations-II

Time: 3 Hours Max. Marks:100

General Instructions:

IMP: Verify that you have received question paper with correct course, code, branch etc.

1. *This Question paper comprises of three Sections -A, B, & C. It consists of Multiple Choice Questions (MCQ's) & Subjective type questions.*
2. *Maximum marks for each question are indicated on right hand side of each question.*
3. *Illustrate your answers with neat sketches wherever necessary.*
4. *Assume suitable data if necessary.*
5. *Preferably, write the answers in sequential order.*
6. *No sheet should be left blank. Any written material after a blank sheet will not be evaluated/checked.*

Section A

1-Attempt all parts:-

1-a Beta function $B(1,1) = a$. Then (CO1) 1

(i) $a = 1$ (ii) $a = 2$

(iii) $a = 3$ (iv) $a = 0$

1-b If $I = \int_0^1 \int_0^x 2dxdy$. Then (CO1) 1

(i) $I = 0$ (ii) $I = 1$

(iii) $I = 1.5$ (iv) $I = 2$

1-c The complementary function of $\frac{d^2y}{dx^2} + y = 0$ is (CO2) 1

(i) $(c_1 \cos x + c_2 \sin x)$ (ii) $(c_1 + c_2)\sin x$

(iii) $c_1 e^{x} + c_2 e^{-x}$ (iv) $(c_1 + c_2 x)\cos x$

1-d The particular integral of $\frac{d^4y}{dx^4} + \frac{d^3y}{dx^3} + \frac{d\ y}{dx} - y + 1 = 0$ is (CO2) 1

(i) -1 (ii) $+1$

(iii) $+2$ (iv) -2

1-e Solution of $\frac{\partial z}{\partial x} + \frac{\partial z}{\partial y} = 1$ is (CO3) 1

(i) $f(xy, y-z) = 0$ (ii) $f(x-y, y+z) = 0$

(iii) $f(x+y, y-z) = 0$ (iv) $f(x-y, z-y) = 0$

1-f PDE: $\frac{\partial u}{\partial t} - a\frac{\partial^2 u}{\partial x^2} = 0, a$ is positive constant. The given PDE is (CO3) 1

(i) Hyperbolic (ii) Elliptic

(iii) Parabolic (iv) None of these

1-g If $I = L\left(\int_0^t f(t)dt\right)$, where $L\{f(t)\} = F(s)$. Then (CO4) 1

(i) $I = \frac{F(s^2)}{s^2}$ (ii) $I = \frac{1}{s}F\left(\frac{1}{s}\right)$

(iii) $I = sF\left(s\right)$ (iv) $I = \frac{F(s)}{s}$

1-h Inverse Laplace transform of $\frac{1}{(s-a)^2 + b^2}$ is (CO4) 1

(i) (i) $\frac{1}{b}e^{-at}\sin bt$ (ii) $\frac{1}{b}e^{at}\sin bt$

(iii) $\frac{1}{b}e^{at}\cos bt$ (iv) $\frac{1}{b}e^{-at}\cos bt$

1-i Pointing to a photograph of a boy Suresh said, "He is the son of the only son of my mother." How is Suresh related to that boy? (CO5) 1

(i) Brother (ii) Uncle (iii) Cousin (iv) Father

1-j-A sum of Rs. 2000 amounts to Rs. 4000 in two years at compound interest. In how many years does the same amount becomes Rs. 8000? (CO5) 1

(i) 6 (ii) 8 (iii) 2 (iv) 4

2- Attempt all parts:-

2.a. Evaluate $\int_0^1 \left(1-x^3\right)^{\frac{-1}{2}} dx$ (CO1) 2

2.b Solve $\frac{d^2y}{dx^2} + y = sinx$ (CO2) 2

2.c. Solve $x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} = z$. (CO3) 2

2.d. Find $L^{-1}(\frac{1}{s(s+5)})$ (CO4) 2

2.e Two numbers are in the ratio 3:5. If 9 be subtracted from each, then they are in the ratio of 12:23. Find the second number. (CO5) 2

Section B

3- Answer any five of the following-.

3.a Show that $\int_0^{\frac{\pi}{2}} \sin^p\theta \cos^q\theta d\theta = \frac{\Gamma\left(\frac{p+1}{2}\right)\Gamma\left(\frac{q+1}{2}\right)}{2\Gamma\left(\frac{p+q+2}{2}\right)}$. (CO1) 6

3.b. Evaluate $\int_0^1 \int_y^1 \frac{x}{x^2+y^2} dxdy$ by changing the order of integration . (CO1) 6

3.c. Solve using variation of parameter $\frac{d^2y}{dx^2} + y = \sec x$ (CO2) 6

3.d. Solve $(y+z)\frac{\partial z}{\partial x} + (x+z)\frac{\partial z}{\partial y} = (x+y)$ (CO3) 6

3.e .If $f(t) = \frac{(1-cos2t)}{t^2}$, then find $L\{f(t)\}$. (CO4) 6

3.f. Evaluate $L^{-1}\left[\frac{2s+1}{(s-1)^2(s+2)^2}\right]$. (CO4) 6

3.g Gopal starts from his house towards West. After walking a distance of 30 m, he turned towards right and walked 20 m. He then turned left and moving a distance of 10 m, turned

to his left again and walked 40 m. He now turns to the left and walks 5 m. finally he turns to his left. In which direction is he walking now?

(i) Ravi's age after 6 year will be three-seventh of his father's age. Ten years ago, the ratio of their ages was 1:5. What is Ravi's father's age at present? (CO5) 6

Section C

4- Answer any one of the following-

4a. Apply Dirichlet's integral to find the volume and mass of the solid bounded by the ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$,the density at any point being $\rho = kxyz$. (CO1) 10

(CO1)

4b. Evaluate $\int\int_A x^2 dxdy$, where A is the region in the first quadrant bounded by the curves $xy=16$, $x=y, y=0, x=8$. (CO1) 10

5- Answer any one of the following-

5a. Solve : $x^2\frac{d^2y}{dx^2}+4x\frac{dy}{dx}+2y=e^x+logx$. (CO2) 10

5b. Solve simultaneous differential equations: $\frac{dx}{dt}+\frac{dy}{dt}+3x=sint$ and

$\frac{dx}{dt}+y-x=cost$. (CO2) 10

6- Answer any one of the following-

6a. Solve: $(D^2-DD'+D'-1)z=\cos(x+2y)+e^y$. (CO3) 10

6b. Solve: $(D^2-6DD'+9D'^2)z=12x^2+36xy$. (CO3) 10

7- Answer any one of the following-

7a. State convolution theorem , and find $L^{-1}\left[\frac{s^2}{(s^2+a^2)(s^2+b^2)}\right]$ by using convolution theorem. (CO4) 10

7b. Solve the initial value problem $2\frac{d^2y}{dt^2}+5\frac{d\ y}{dt}+2y=e^{-2t}, y=0, \frac{d\ y}{dt}=0 \text{ at } t=0$ using Laplace transform. (CO4) 10

8- Answer any one of the following-

8a. (i) The milk and water in two vessel A and B are in the ratio 4:3 and 2:3 respectively. In what ratio, the liquids in both the vessels be mixed to obtain a new mixture in vessel C containing half milk and half water?

(ii) 60 kg of an alloy A is mixed with 100kg of alloy B. If alloy A has lead and tin in the ratio 3:2 and alloy B has tin and copper in the ratio 1:4 . Find the amount of tin in the new alloy. (CO5) 10

8b. (i) Five years ago, the ratio of Alice's age to Bob's age was 3:2. The ratio of their ages five years from now will be 4:3. Determine their current ages.

(ii) In a family gathering, there are six people - A, B, C, D, E, and F. A is the father of B and D. C is the sister of B. D is married to E. F is the daughter of E. What is the relationship between C and F? (CO5) 10

End of the question paper