Printed page: 4

Subject Code: AMIAS0203

Roll No: | | | | | | | | | | | | | |

NOIDA INSTITUTE OF ENGINEERING AND TECHNOLOGY, GREATER NOIDA

(An Autonomous Institute Affiliated to AKTU, Lucknow)

M.Tech (Integrated)

SEM: II- THEORY EXAMINATION (2022-2023)

Subject: Engineering Mathematics - II

Time: 3 Hours Max. Marks: 100

General Instructions:

IMP: Verify that you have received question paper with correct course, code, branch etc.

1. *This Question paper comprises of three Sections -A, B, & C. It consists of Multiple Choice Questions (MCQ's) & Subjective type questions.*
2. *Maximum marks for each question are indicated on right hand side of each question.*
3. *Illustrate your answers with neat sketches wherever necessary.*
4. *Assume suitable data if necessary.*
5. *Preferably, write the answers in sequential order.*
6. *No sheet should be left blank. Any written material after a blank sheet will not be evaluated/checked.*

SECTION – A 20

1. Attempt all parts:-

1-a. The following differential equation has $3\frac{d^2y}{dx^2} + 3(\frac{dy}{dx})^3 + y^2 + 2 = x^5$ (CO1) 1

(a) degree=2, order=1

(b) degree=1, order=2

(c) degree=3, order=1

(d) None of these

1-b. A solution of the following differential equation is given by (CO1) 1

$$\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 6y = 0$$

(a) $y = c_1e^{2x} + c_2e^{-3x}$

(b) $y = c_1e^{2x} + c_2e^{3x}$

(c) $y = c_1e^{-2x} + c_2e^{-3x}$

(d) None of these

1-c. State, which one of the alternatives is correct : The series 1-1+1-1+1…… is (CO2) 1

(a) Convergent with its sum 0

(b) Convergent with its sum 1

(c) Divergent

(d) Oscillatory

1-d. The values of a_0 and a_n for the function $f(x) = x^3$ in the interval $-\pi < x < \pi$ are (CO2) 1

(a) 0, 0

(b) 1, 2

(c) 0, 3

(d) None of these

1-e. If L{F(t)} = f(s), then L{F(at)} is (CO3) 1

(a) $\frac{1}{a} f(\frac{s}{a})$

(b) $\frac{1}{a} f(s-a)$

(c) $f(\frac{s}{a})$

(d) None of these

1-f. Laplace transform of Unit Step function $u(t-a)$ is (CO3) 1

(a) $\frac{1}{s}e^{as}$

(b) $\frac{1}{s}e^{-as}$

(c) e^{as}

(d) None of these

1-g. If $\vec{v} = yzi + 3\,xzj + zk$, then $curl\ \vec{v}$ is (CO4) 1

(a) $3xi - yj + 2zk$

(b) $-3xi + yj + 2zk$

(c) $3xi - yj - 2zk$

(d) None of these

1-h. A vector field with vanishing curl is called an (CO4) 1

(a) Rotational vector

(b) Irrotational vector

(c) Solenoidal vector

(d) None of these

1-i. If $a : b = 5 : 9$ and $b : c = 7 : 4$ then $a : b : c$ is (CO5) 1

(a) $35 : 63 : 36$

(b) $36 : 63 : 36$

(c) $35 : 33 : 36$

(d) None of these

1-j. Ravi lent Rs. 5000 to Santosh for 3 years at the rate of 5% per annum compound interest. Calculate the amount that Ravi will get after 3 years. (CO5) 1

a) 5788

b) 5788.125

c) 5788.135

d) None of these

2. Attempt all parts:-

2.a. The particular integral (PI) of the differential equation (CO1) 2

$x^2\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + 2y = 4$ is…… .

2.b. Test the convergence of the series $\frac{4}{1} + \frac{5}{4} + \frac{6}{9} + \frac{7}{16} + \cdots$ (CO2) 2

2.c. Find the Laplace transform of $\{e^{2t}t^2\}$. (CO3) 2

2.d. Write the statement of Gauss Divergence theorem. (CO4) 2

2.e. In what ratio must a shopkeeper mix two types of rice worth Rs. 50 kg and Rs. 70 kg, so that the average cost of the mixture is Rs. 65 kg? (CO5) 2

SECTION – B 30

3. Answer any <u>five</u> of the following-

3-a. Solve the differential equation $(D-2)^2y = e^{3x}sin2x$. (CO1) 6

3-b. Solve the simultaneous differential equation $\frac{dx}{dt} + 5x - 2y = t, \frac{dy}{dt} + 2x + y = 0$. (CO1) 6

3-c. Test the convergence of the series $\frac{1}{1.2.3} + \frac{1}{2.3.4} + \frac{1}{3.4.5} + \cdots \ldots$. (CO2) 6

3-d. Obtain the half range sine and cosine series for the function $f(x) = x \ \ in\ 0 < x < 2$. (CO2) 6

3-e. Find the Laplace transform of $\int_0^t e^t \frac{sint}{t}\, dt$. (CO3) 6

3-f. Find the directional derivative of the function $\phi = x^2 - y^2 + 2z^2$ at the point P(1, 2, 3) in the direction of the line PQ where Q is the point (5, 0, 4). (CO4) 6

3-g. The product of the ages of Aaisha and Niti is 540. If twice the age of Aaisha is more than Niti's age by 6 years, then find Aaisha's age? (CO5) 6

SECTION – C 50

4. Answer any <u>one</u> of the following-

4-a. Solve the differential equation $cosx\frac{d^2y}{dx^2} + sinx\frac{dy}{dx} - 2ycos^3x = 2cos^5x$ by changing into normal form. (CO1) 10

4-b. Solve the differential equation $\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = \frac{e^x}{1+e^x}$ by the method of variation of parameters. (CO1) 10

5. Answer any <u>one</u> of the following-

5-a. Test the convergence of the following series: 10

$1 + \frac{3}{7}x + \frac{3.6}{7.10}x^2 + \frac{3.6.9}{7.10.13}x^3 + \cdots \ldots; x > 0.$ (CO2)

5-b. Find the Fourier series of the periodic function f with period 2π defined as follows: 10

$f(x)=\begin{cases} 0, & for\ -\pi < x \le 0, \\ x, & for\ 0 \le x \le \pi \end{cases}$. Hence prove that $1+\frac{1}{3^2}+\frac{1}{5^2}+\frac{1}{7^2}+...=\frac{\pi^2}{8}$. (CO2)

6. Answer any one of the following-

6-a. Use convolution theorem to evaluate $L^{-1}\left\{\frac{s}{(s^2+9)^2}\right\}$. (CO3) 10

6-b. Solve the differential equation using Laplace transform $y'' + 2y' + y = te^{-t}$, if $y(0) = 1, y'(0) = 2$. (CO3) 10

7. Answer any one of the following-

7-a. Verify Stoke's theorem for $\vec{F} = (x^2 + y^2)i - 2\,xyj$ taken round the rectangle bounded by the lines $x = \pm a, y = 0. y = b$. (CO4) 10

7-b. Apply Green's theorem to evaluate $\int[(2x^2 - y^2)dx + (x^2 + y^2)dy]$ over the region C, where C is the boundary of the area enclosed by the x-axis and the upper half of circle $x^2 + y^2 = a^2$. (CO4) 10

8. Answer any one of the following-

8-a. (i) The population of a town decreased every year due to migration, poverty and unemployment. The present population of the town is 6,31,680. Last year the migration was 4%, and the year before last, it was 6%. What was the population two years ago? 10

(ii) Sanjeev walks 10 m towards the South. Turning to the left, he walks 20 m and then moves to his right. After moving a distance of 20 m, he turns to the right and walks 20 m. Finally, he turns to the right and moves a distance of 10 m. How far and in which direction is he from the starting point? (CO5)

8-b. (i) A container contains a mixture of two liquids P and Q in the ratio of 7:5. When 9 liters of mixture is taken out and replaced with Q, the ratio becomes 7:9. Find the quantity of liquid P in the container. 10

(ii) A container contains 40 litres of milk. From this container, 4 litres of milk was taken out and replaced by water. This process was repeated further two times. How much milk is now contained by die container? (CO5)