NOIDA INSTITUTE OF ENGINEERING AND TECHNOLOGY, GREATER NOIDA
(An Autonomous Institute Affiliated to AKTU, Lucknow)
B.Tech
SEM: I - THEORY EXAMINATION (2024 - 2025)
Subject: Engineering Mathematics-I

Time: 3 Hours   Max. Marks: 100

General Instructions:

IMP: *Verify that you have received the question paper with the correct course, code, branch etc.*
*1. This Question paper comprises of **three Sections -A, B, & C.** It consists of Multiple Choice Questions (MCQ's) & Subjective type questions.*
2. Maximum marks for each question are indicated on right -hand side of each question.
3. Illustrate your answers with neat sketches wherever necessary.
4. Assume suitable data if necessary.
5. Preferably, write the answers in sequential order.
6. No sheet should be left blank. Any written material after a blank sheet will not be evaluated/checked.

SECTION-A   20

1. Attempt all parts:-

1-a.   1

If the product of Eigen values of matrix $A = \begin{bmatrix} 1 & 2 & -1 \\ 3 & 5 & 2 \\ 1 & k & 2 \end{bmatrix}$ is -8 . Then the value of k will be   (CO1,K1)

(a)   3

(b)   2

(c)   -2

(d)   -3

1-b.   1

If a matrix of order *m x n* can be expressed as $A = \begin{bmatrix} I_r & 0 \\ 0 & 0 \end{bmatrix}$, the rank of A is (CO1,K2)

(a)   r

(b)   r+1

(c)   r-1

(d)   none of these

1-c.   1

If $f(x,y) = (x^{\frac{1}{4}} - y^{\frac{1}{4}})(3x^{\frac{1}{5}} - y^{\frac{1}{5}})$ then the value of $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}$ is (CO2,K2)

(a)

$$\frac{9}{20}f$$

(b) $-\frac{9}{20}f$

(c) $-\frac{1}{20}f$

(d) $\frac{1}{20}f$

1-d. If $y = \sin(m\sin^{-1}x)$ then after two time differentiation we get (CO2,K3)   1

(a) $(1-x^2)y_2 - xy_1 - m^2 y = 0$

(b) $(1-x^2)y_2 - xy_1 + m^2 y = 0$

(c) $(1-x^2)y_2 + xy_1 + m^2 y = 0$

(d) None of these.

1-e. If E = RI and possible error in E and I are 20% and 10%, then percentage error in   1
R is (CO3,K2)

(a) 10%

(b) 15%

(c) 30%

(d) 20%

1-f. Expannsion of $f(x,y)$ at $(0,0)$ is given by (CO3,K1)   1

(a) $f(0, 0) + xf'(0, 0) + xf'(0, 0) + x^2 f''(0, 0) + y^2 f''(0, 0) + ...$

(b) $f(0, 0) + xf_x(0, 0) + yf_y(0, 0) + \frac{1}{2!}\left[x^2 f_{x^2}(0, 0) + y^2 f_{y^2}(0, 0) + 2xyf_{xy}(0, 0)\right] + ...$

(c) $f(0, 0) + xf_x(0, 0) + yf_y(0, 0) + y^2 f_{x^2}(0, 0) + x^2 f_{y^2}(0, 0) + ...$

(d) None of these

1-g.   1

By changing order $I = \int_0^a \int_y^a f(x,y)\,dxdy$ leads to the value $I = \int_r^s \int_p^q f(x,y)\,dydx$.
What is p ? (CO4,K3)

(a) y

(b) a

(c) o

(d) None of these

1-h. The value of $\Gamma(-1/2)$ is (CO4,K2)   1

(a) $2\sqrt{\pi}$

(b) $-2\sqrt{\pi}$

(c) $-\sqrt{\pi}$

(d) None of these

1-i. In an election contest between A and B ,A wins by the margin of 240 votes . If A get 60 % of the total votes , total vote are (CO5,K1)   1

(a) 1000

(b) 900

(c) 800

(d) 1200

1-j. In a certain code "RANGE" is coded as 12345 and "RANDOM" is coded as 123678. Then the code for the word "MANGO" would be (CO5,K2)   1

(a) 82357

(b) 84629

(c) 82347

(d) 83274

2. Attempt all parts:-

2.a. Find the value of μ such that the rank of the matrix $\begin{bmatrix} \mu & -1 & 0 \\ 0 & \mu & -1 \\ -1 & 0 & \mu \end{bmatrix}$ is equal to 2. (CO1,K2)   2

2.b. Find the nth derivative of $x^2 \sin x$ at $x = 0$. (CO2,K3)   2

2.c. Determine the points where the function $u = x^2 + y^2 + 6x + 12$ has a maximum or minimum. (CO3,K1)   2

2.d. Evaluate the value of $\int_0^1 x^4(1-x)^3 \, dx$. (CO4,K1)   2

2.e. What is a single discount equivalent to three successive discounts of 5%, 10%, 20%? (CO5,K3)   2

SECTION-B   30

3. Answer any <u>five</u> of the following:-

3-a. If $A = \begin{bmatrix} -1 & 2+i & 5-3i \\ 2-i & 7 & 5i \\ 5+3i & -5i & 2 \end{bmatrix}$. Show that A is hermitian matrix and iA is skew-Hermitian matrix. (CO1,K1)   6

3-b. Find the inverse of the matrix $\begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}$ by elementary transformation. (CO1,K2)   6

3-c. If $u = e^{xyz}$, then show that $\dfrac{\partial^3 u}{\partial x \, \partial y \, \partial z} = (1 + 3xyz + x^2 y^2 z^2) \, e^{xyz}$. (CO2,K2)   6

3-d. If $u = \log(\tan x + \tan y + \tan z)$, prove that $\sin 2x \dfrac{\partial u}{\partial x} + \sin 2y \dfrac{\partial u}{\partial y} + \sin 2z \dfrac{\partial u}{\partial z} = 2$. (CO2,K1)   6

3.e. If $u = x + 2y + z, v = x - 2y + 3z$ and $w = 2xy - xz + 4yz - 2z^2$, show that they   6

are not independent. Find the relation between *u, v and w.*   (CO3,K2)

3.f.   Change into polar co-ordinates and hence evaluate the integral   6
$$\int_0^\infty \int_0^\infty e^{-(x^2+y^2)} dy\, dx$$.   (CO4,K2)

3.g.   In certain code language 'si po re' means ' book is thick ', 'ti na re' means 'bag is heavy', 'ka si' means ' interesting book' and 'de ti' means 'that bag'. What should stand for ' that is interesting'in that code language?   (CO5,K1)   6

SECTION-C   50

4. Answer any <u>one</u> of the following:-

4-a.   Verify Cayley-Hamilton theorem for the matrix $A=\begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$ and hence compute A^{-1} . Also evaluate $A^6-6A^5+9A^4-2A^3-12A^2+23A-9I$. (CO1,K3) .   10

4-b.   Find the eigen values and eigen vectors of a matrix $\begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$. (CO1,K2)   10

5. Answer any <u>one</u> of the following:-

5-a.   If $u=f(2x-3y, 3y-4z, 4z-2x)$, then prove that $\frac{1}{2}\frac{\partial u}{\partial x}+\frac{1}{3}\frac{\partial u}{\partial y}+\frac{1}{4}\frac{\partial u}{\partial z}=0$ 10
(CO2,K2)

5-b.   If $y = \sin(a \sin^{-1} x)$, then find $y_n(0)$. (CO2,K3)   10

6. Answer any <u>one</u> of the following:-

6-a.   Find the dimension of the rectangular box of maximum capacity whose surface area is given when
(i) Box is open at the top
(ii) Box is closed (CO3,K2)   10

6-b.   Expand the function $\tan^{-1}\left(\frac{y}{x}\right)$ in the neighborhood of (1, 1) up to and inclusive of second degree terms. Hence compute the value of the function at (1.1, 0.9) approximately.   (CO3,K2)   10

7. Answer any <u>one</u> of the following:-

7-a.   Apply Dirichlet theorem to evaluate mass of the tetrahedron OABC for the plane $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$, that meets the coordinate axis at A,B and C, if the density at a point is $\rho(x,y,z) = kxyz$.   (CO4,K1)   10

7-b.   Change the order of integration for $I=\int_0^1 \int_{x^2}^{2-x} xy\, dy\, dx$ and evaluate the same.   (CO4,K3)   10

8. Answer any <u>one</u> of the following:-

8-a. (a) Rakesh bought a cycle for Rs. 800. and marked it up by 50% and gave a discount of 10%. A customer came in to buy the cycle and bargained with Rakesh for an additional discount of 20% on the already discounted price. What is the profit earned by Rakesh?   10

(b) If in a code language, SPARK is written as TQBSL, then what will be the code for FLAME in that language ?

(c) The average of five numbers is 56. If the average of first four numbers is 54, what is the value of the fifth number ? (CO5,K2)

8-b. (a) In an examination, 50% students failed in English and 40% in Math and 15% students failed in both subjects. If 200 students passed in both the subjects, find the number of students appeared in the examination ?   10

(b) In a certain code 'MISSIONS' is written as 'MSIISNOS'. How is 'ONLINE' written in that code?

(c) A person bought an article and sold it at a loss of 10%. If he had bought it for 20% less and sold it for Rs.55 more, he would have had a profit of 40%.Find the C.P. of the article? (CO5,K3)