Printed page: …. **Subject Code: BAS0104**

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JIDA INSTITUTE OF ENGINEERING AND TECHNOLOGY, GREATER NOIDA

(An Autonomous Institute Affiliated to AKTU, Lucknow)

B.Tech

SEM:-I - THEORY (2024-2025)

Subject Mathematical Foundations-I

Time: 3 Hours **Max. Marks:100**

General Instructions:

IMP: Verify that you have received question paper with correct course, code, branch etc.

1. *This Question paper comprises of three Sections -A, B, & C. It consists of Multiple Choice Questions (MCQ's) & Subjective type questions.*
2. *Maximum marks for each question are indicated on right hand side of each question.*
3. *Illustrate your answers with neat sketches wherever necessary.*
4. *Assume suitable data if necessary.*
5. *Preferably, write the answers in sequential order.*
6. *No sheet should be left blank. Any written material after a blank sheet will not be evaluated/checked.*

Section A

1. Answer all "objective/one-word answer" type questions. Choose the correct option from the four given options in objective type questions; only one of these options is correct.

(a) If A is skew-Hermitian matrix then iA is (CO1,k2)

(i) Symmetric

(ii) Skew-symmetric.

(iii) Hermitian.

(iv) Skew-Hermitian.

(b) Product of eigenvalues of a matrix is equal to (CO1,K1)

(i) the transpose of a matrix

(ii) the determinant of a matrix

(iii) the inverse of a matrix

(iv) None of these

(c) The dimension of vector space R^3 is (CO2,K1)

(i) 2

(ii) 3

(iii) 1

(iv) None of these

(d) The vectors (4,3)and (8,6) are (CO2,K2)

(i) Linearly Dependent

(ii) Linearly Independent

(iii) Both (i) and (ii)

(iv) None of these

(e) 1001^{th} derivative of $(x+1)^{1000}$ is (CO3,K2)

(i) 0

(ii) $1000x$

(iii) $1000x^{999}$

(iv) 1000!

(f) If $F(x,y,z) = x^2 + xyz + z$.Find $\frac{\partial f}{\partial x}$ at (1,1,1) (CO3,K2)

(i) 0

(ii) 1

(iii) 3

(iv) -1

(g) The percentage error in the area of a rectangle when an error of +1 percent is made in measuring its length and breadth is (CO4,K2)

(i) 2%

(ii) 3%

(iii) 4%

(iv) 5%

(h) If $u = x^2$ and $v = y^2$, then the value of $\frac{\partial(u,v)}{\partial(x,y)}$ is (CO4,K2)

(i) 4xy

(ii) 2xy

(iii) 6xy

(iv) 8xy

(i) If Reeta purchased a refrigerator for 15,000 rupees and spent 500 rupees on installation, then sold it for 16,800 rupees, what is her profit? (CO5,K2)

(i) Rs.1000

(ii) Rs.1200

(iii) Rs.1300

(iv) None of these

(j) A man had 7 children. When their average age was 12 years, a child aged 6 years died. The average of remaining 6 children is (CO5,K2)

(i) 10 years

(ii) 13 years

(iii) 11 years

(iv) 14 years

2. Very short answer type questions each carrying 2 marks.

(a) Write the characteristic equation of $\begin{bmatrix} 69 & 44 & -9 \\ 0 & -98 & 43 \\ 0 & 0 & -198 \end{bmatrix}$. (CO1,K2)

(b) Define the term vector space. (CO2,K1)

(c) Evaluate $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}$ if $u = \frac{x^3+y^3}{x^2+y^2}$. (CO3,K1)

(d) What is the general form of the Taylor series expansion of a function $f(x)$ about $x = a$. (CO4,K1)

(e) Find the value of x in the following number series: $1, 8, 27, x, 125$. (CO5,K2)

Section B

3. Attempt any five questions. Each carrying 6 marks.

(a) Find the rank of A by reducing it into normal form,

$$A=\begin{bmatrix} 2 & 3 & -1 & -1 \\ 1 & -1 & -2 & -4 \\ 3 & 1 & 3 & -2 \\ 6 & 3 & 0 & -7 \end{bmatrix}.$$ (CO1,K3)

(b) Find the eigen values and eigen vectors of the matrix $A = \begin{bmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{bmatrix}$. (CO1,K3)

(c) Show that the vectors (1, 2, 1), (2, 1, 0), (1, -1, 2) form a basis of R^3. (CO2,K2)

(d) If α *and* β are vectors in an inner product space then show that $\|\alpha+\beta\|^2 + \|\alpha-\beta\|^2 = 2\|\alpha\|^2 + 2\|\beta\|^2$. (CO2,K3)

(e) If $u = \sin^{-1}\left(\frac{x^{1/3}+y^{1/3}}{x^{1/2}+y^{1/2}}\right)^{1/2}$, then evaluate $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}$ and $x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2}$. (CO3,K3)

(f) Verify whether the following functions are functionally dependent. If they are, find the relation between them. Where $u = \frac{x+y}{1-xy}, v = \tan^{-1} x + \tan^{-1} y.$ (CO4,K3)

(g) In certain code language 'si po re' means 'book is thick', 'ti na re' means 'bag is heavy', 'ka si' means 'interesting book' and 'de ti' means 'that bag'. What should stand for 'that is interesting' in that code language? (CO5,K3)

Section C

This section consists of five questions (Question 4 to 8) having two parts *a* and b each carrying 10 marks. You are required to attempt any one part *a* or b.

4a. Determine the value of λ and μ so that the equations $2x + 3y + 5z = 9, 7x + 3y - 2z = 8, 2x + 3y + \lambda z = \mu$, have (i) no solution, (ii) a unique solution and (iii) infinite many solutions. (CO1,K3)

4b. Verify Cayley-Hamilton theorem for $A = \begin{bmatrix} 1 & 0 & -1 \\ 1 & 2 & 1 \\ 2 & 2 & 3 \end{bmatrix}$. Also find A^{-1} if exists. (CO1,K3)

5a. Show that the mapping $T: R^2 \to R^3$ defined as $T(x, y) = (x - y, y - x, -x)$ is a linear transformation. Find the range, null-space and nullity of T. (CO2,k3)

5b. Find the basis and dimension of the subspace W of R^4 generated by (1, -4, -2, 1), (1, -3, -1, 2), (3, -8, -2, 7). (CO2,K3)

6a. . If $y = \left[x + \sqrt{1+x^2}\right]^m$, find $y_n(0)$. (CO3,K3)

6b. If $u = f(2x-3y, 3y-4z, 4z-2x)$ then prove that $\frac{1}{2}\frac{\partial u}{\partial x} + \frac{1}{3}\frac{\partial u}{\partial y} + \frac{1}{4}\frac{\partial u}{\partial z} = 0$.(CO3,K3)

7a. Find the dimension of the rectangular box of maximum capacity whose surface area is given when

(i) Box is open at top
(ii) Box is closed. (CO4,K3)

7b. Expand $e^x cosy$ in the power of $(x-1)$ and $\left(y - {}^{\pi}/_{4}\right)$ upto third degree terms. (CO4,K3)

8a. (i) Instead of selling the bicycle for Rs.2000, a shopkeeper sold it for Rs. 1500.Find the loss incurred in transaction ?

(ii) In a school with 600 students, the average age of the boys is 12 years and that of the girls is 11 years. If the average age of the school is 11 years and 9 months, then find the number of girls in the school?

(iii) Water tax is increased by 20% but its consumption is decreased by 20%.Find the increase or decrease in the expenditure of the money?(CO5,K3)

8b. (i) If the price of an item is decreased by 10% and then increased by 10%, find the net effect on the price of the item?

(ii) A candidate scores 25 % marks and fails by 30 marks, while another candidate who scores 50 % marks get 20 marks more than the minimum marks required to pass the examinations. Find the maximum marks for the examination?

(iii) The averages of runs scored by cricket player in 11 innings is 63 and the average of his first six innings is 60 and the average of last six innings is 65. Find the runs scored by him in the sixth innings? (CO5,K3)