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NOIDA INSTITUTE OF ENGINEERING AND TECHNOLOGY, GREATER NOIDA

(An Autonomous Institute Affiliated to AKTU, Lucknow)

B. Tech

SEM II THEORY EXAMINATION 2025-2026

Subject: Differential Equations and Linear Transformations

Time: 3 Hours

Max. Marks: 100

General Instructions:

IMP: Verify that you have received question paper with correct course, code, branch etc.

1. This Question paper comprises of three Sections -A, B, & C. It consists of Multiple Choice Questions (MCQ's) & Subjective type questions.
2. Maximum marks for each question are indicated on right hand side of each question.
3. Illustrate your answers with neat sketches wherever necessary.
4. Assume suitable data if necessary.
5. Preferably, write the answers in sequential order.
6. No sheet should be left blank. Any written material after a blank sheet will not be evaluated/checked.

SECTION – A

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1. Attempt all parts:-

- (1.1) The Complementary function of the differential equation $(D - 2)^3 = 0$ is c : 1
- (a) $(a + bx + cx^2)e^{2x}$ (CO1,K2)
- (b) $(a + bx - cx^2)e^{2x}$
- (c) $(a - bx + cx^2)e^{2x}$
- (d) None of these
- (1.2) The differential equation $\frac{d^4y}{dx^4} + 13\frac{d^2y}{dx^2} + 36y = 0$ is : (CO1,K1) 1
- (a) Order 4 and degree 1
- (b) Order 4 and degree 2
- (c) Order 1 and degree 4
- (d) None of these
- (1.3) The functions $\sin nx$ and $\cos nx$ are periodic with period : (CO2,K1) 1
- (a) $2\pi/n$
- (b) π/n
- (c) $x\pi/n$
- (d) None of these

- (1.4) The general half range sine series of function $f(x)$ in the $(0, l)$ can be given as : 1
 (a) $f(x) = \sum_1^\infty b_n \sin \frac{n\pi x}{l}$ (CO2, K1)
 (b) $f(x) = \sum_1^\infty b_n \sin nx$
 (c) $f(x) = \sum_1^\infty b_n \sin \frac{\pi x}{l}$
 (d) None of these
- (1.5) If Laplace transform of $\{F(t)\} = f(s)$, then $L\{F(at)\}$ is (CO3,K1) 1
 (a) $1/a f(s/a)$
 (b) $\frac{1}{a} f(s - a)$
 (c) $f(\frac{s}{a})$
 (d) None of these
- (1.6) Laplace transform of $f(t) = 5e^{2t} + 2$ is : (CO3,K2) 1
 (a) $\frac{5}{s-2} - \frac{1}{s}$
 (b) $\frac{1}{s-2}$
 (c) $\frac{5}{s-2} + \frac{2}{s}$
 (d) $\frac{1}{s-2} + \frac{1}{s}$
- (1.7) The vectors $(1, 0, 0)$, $(0, 1, -1)$ and $(0, 1, 2)$ are: (CO4,K2) 1
 (a) Linearly dependent
 (b) Linearly independent
 (c) (a) & (b) both
 (d) None of these
- (1.8) Which of the following does not form a basis for R^2 ? (CO4, K2) 1
 (a) $(1,0), (0,1)$
 (b) $(1,1), (2,2)$
 (c) $(1,2), (3,4)$
 (d) $(5,0), (0,3)$
- (1.9) Let $T: R^5 \rightarrow R^3$ be a linear transformation. If the rank of is 2, what is the nullity of T ? (CO5, K2) 1
 (a) 1
 (b) 2
 (c) 3
 (d) None of these
- (1.10) If α, β are two orthogonal unit vectors in an inner product space $V(F)$, then $\|\alpha + \beta\|$ is: (CO5, K2) 1
 (a) 0
 (b) $\sqrt{2}$

- (c) 1
(d) None of these

2. Attempt all parts:-

- (2.1) Find the P.I. of differential equation $y'' + a^2y = \sin ax$. (CO1,K2) 2
(2.2) Find the coefficient a_0 in a Fourier series for the Function $f(x) = x$ in the interval $0 < x < 2\pi$. (CO2,K3) 2
(2.3) Find value of the integral $\int_0^\infty \frac{\sin t}{t} dt$. (CO3, K3) 2
(2.4) Define Basis and Dimension of a Vector space. (CO4,K1) 2
(2.5) Define Rank and Nullity theorem. (CO5,K1) 2

SECTION – B

30

Answer any one of the following-

- (3.1) Solve the simultaneous differential equation $\frac{dx}{dt} + 5x - 2y = t, \frac{dy}{dt} + 2x + y = 0$. (CO1,K3) 6
(3.2) Solve the ordinary differential equation: $x^2 \frac{d^2y}{dx^2} - 4x \frac{dy}{dx} + 6y = x^2$. (CO1,K3) 6

Answer any one of the following-

- (3.3) Solve the PDE : $x(y^2 + z)p - y(x^2 + z)q = z(x^2 - y^2)$. (CO2,K3) 6
(3.4) Find the half range cosine series expansion of $f(x) = x - x^2$ in $0 < x < 1$. (CO2,K3) 6

Answer any one of the following-

- (3.5) Express the following function in terms of unit step function and Find the Laplace transform of the function: $f(t) = \begin{cases} 2, & \text{if } 0 < t < \pi \\ 0, & \text{if } \pi < t < 2\pi \\ \sin t, & \text{if } t > 2\pi \end{cases}$ (CO3,K3) 6
(3.6) Find the Laplace Transform of $t^2 e^t \sin t$. (CO3, K3) 6

Answer any one of the following-

- (3.7) If $V = \{(x, y, z) : x, y, z \in R\}$ where R is the field of real numbers then show that the set $W = \{(x, y, z) : x - 3y + 4z = 0\}$ is a subspace of V over R . (CO4, K3) 6
(3.8) Determine a basis and dimension of the subspace spanned by the vectors $a_1 = (1,0,0), a_2 = (0,2,0), a_3 = (0,0,3)$ and $a_4 = (1,2,3)$. (CO4, K3) 6

Answer any one of the following-

- (3.9) Let $T : V_2(R) \rightarrow V_2(R)$ is defined by $T(x, y) = (3x + 2y, 3x - 4y)$. Verify whether T is a linear transformation. Find its rank and nullity. (CO5, K3) 6
(3.10) Describe explicitly a linear transformation from $R^3 \rightarrow R^3$ which has its range and subspace spanned by the vectors $(1,0,-1)$ and $(1,2,2)$. (CO5, K3) 6

SECTION – C

50

4. Answer any one of the following-

- (4.1) Solve the following differential equation by variation of parameter method 10

$$: \frac{d^2 y}{dx^2} + y = \tan x. \quad (\text{CO1,K3})$$
- (4.2) Solve the ODE by changing independent variable $x \frac{d^2 y}{dx^2} - \frac{dy}{dx} + 4x^3 y = x^5$. 10

$$(\text{CO1,K3})$$
5. Answer any one of the following-
- (5.1) Find the Fourier series for $f(x) = x + x^2, -\pi < x < \pi$ Hence deduce that 10

$$\frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} \dots = \frac{\pi^2}{12}.$$

$$(\text{CO2,K3})$$
- (5.2) Solve the linear partial differential equation $\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial x \partial y} - 2 \frac{\partial^2 z}{\partial y^2} = (y - 1)e^x$ 10

$$(\text{CO2,K3})$$
6. Answer any one of the following-
- (6.1) Solve the differential equation by Laplace transform $\frac{d^2 y}{dt^2} - 3 \frac{dy}{dt} + 2y = 4t + e^{3t}$ 10
given that $y(0) = 1, \frac{dy}{dt} = -1$ for $t = 0$. (CO3,K3)
- (6.2) Apply Convolution theorem to evaluate $L^{-1} \left\{ \frac{s^2}{(s^2+1)(s^2+4)} \right\}$. 10
 (CO3,K3)
7. Answer any one of the following-
- (7.1) Let V be the set of all pairs (x, y) of real numbers, and let F be the field of real 10
numbers. Define the addition and scalar multiplication in $(x_1, y_1) + (x_2, y_2) =$
 $(x_1 + x_2, y_1 + y_2)$ and $a(x, y) = (ax, ay)$. Is V , with these operations, a vector space
over the field of real numbers? (CO4, K3)
- (7.2) Are the following vectors linearly dependent? If so find the relation between 10
them: $X_1 = (1, 2, 1), X_2 = (2, 1, 4), X_3 = (4, 5, 6)$ and $X_4 = (1, 8, -1)$. (CO4,K3)
8. Answer any one of the following-
- (8.1) Apply Gram-Schmidt Process to the following sequence of vectors to obtain an 10
orthonormal basis for $R^3(R)$ with the standard inner product : $\begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix} \begin{bmatrix} 8 \\ 1 \\ -6 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}.$
 (CO5,K3)
- (8.2) If α and β are vectors in an inner product space $V(F)$, and $a, b \in F$ then prove 10
that $\text{Re}(\alpha, \beta) = \frac{1}{4} \|\alpha + \beta\|^2 - \frac{1}{4} \|\alpha - \beta\|^2$. (CO5,K3)