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NOIDA INSTITUTE OF ENGINEERING AND TECHNOLOGY, GREATER NOIDA
(An Autonomous Institute Affiliated to AKTU, Lucknow)

B.Tech

SEM: II - THEORY EXAMINATION (2025 - 2026)

Subject: Differential Equations and Fourier Transformations

Time: 3 Hours

Max. Marks: 100

General Instructions:

IMP: Verify that you have received the question paper with the correct course, code, branch etc.

1. This Question paper comprises of **three Sections -A, B, & C**. It consists of Multiple Choice Questions (MCQ's) & Subjective type questions.
2. Maximum marks for each question are indicated on right -hand side of each question.
3. Illustrate your answers with neat sketches wherever necessary.
4. Assume suitable data if necessary.
5. Preferably, write the answers in sequential order.
6. No sheet should be left blank. Any written material after a blank sheet will not be evaluated/checked.

SECTION-A

1. Attempt all parts:-

- (1.1) Degree and order of the differential equation $\sqrt{\left(\frac{dy}{dx}\right)^2 + 3y} = \frac{d^2y}{dx^2}$ is 1
 (CO1,K1)
- (a) Ord = 2, Deg = 2
 (b) Ord = 2, Deg = 1
 (c) Ord = 1 , Deg = 1
 (d) none of these.
- (1.2) General solution of the second order linear differential equation 1
 $\frac{d^2y}{dx^2} - 8\frac{dy}{dx} + 16y = 0$ is : (CO1,K1)
- (a) $(A + Bx)e^{4x}$
 (b) $Ae^{-4x} + Be^{-4x}$
 (c) $A + Be^{4x}$
 (d) $(A + Bx)e^{-4x}$
- (1.3) The Series $1 + \frac{1}{3} + \frac{1}{5} + \frac{1}{7} + \frac{1}{9} + \dots$ is . (CO2, K2) 1
- (a) Convergent
 (b) Divergent
 (c) Oscillatory
 (d) None of these
- (1.4) The coefficient of a_0 in a Fourier series for the function 1
 $f(x) = x + x^3$ in $-\pi < x < \pi$ is

(CO2, K3)

- (a) π
- (b) 2π
- (c) 0
- (d) None of these

(1.5) Laplace of $\frac{d^2y}{dx^2}$ is (CO3,K1) 1

- (a) $s^2\bar{y} - sy(0) - y'(0)$
- (b) $s^2\bar{y} + sy(0) + y'(0)$
- (c) $s^2\bar{y} - sy(0)$
- (d) None of these

(1.6) Inverse Laplace of the function $f(s) = \frac{1}{(s+1)^2}$ is (CO3,K1) 1

- (a) te^t
- (b) $\frac{e^{-t}}{t}$
- (c) te^{-t}
- (d) None of these

(1.7) The Complementary function of partial differential equation $(D - D' - 1)(D - D' - 3)z = 0$ is (CO4,K1) 1

- (a) $C.F. = e^xf_1(y+x) + e^{-3x}f_2(y+x)$
- (b) $C.F. = e^xf_1(y+x) + e^{-3x}f_2(y-x)$
- (c) $C.F. = e^xf_1(y+x) + e^{3x}f_2(y+x)$
- (d) $C.F. = e^xf_1(y-x) + e^{3x}f_2(y+x)$

(1.8) The Auxiliary equation of the partial differential equation $(x^2 - y^2 - z^2)p + 2xyq = 2xz$ is (CO4,K1) 1

- (a) $\frac{dx}{x^2+y^2+z^2} = \frac{dy}{2xy} = \frac{dz}{2xz}$
- (b) $\frac{dx}{x^2-y^2-z^2} = \frac{dy}{2xz} = \frac{dz}{2xy}$
- (c) $\frac{dx}{x^2-y^2-z^2} = \frac{dy}{2xy} = \frac{dz}{2xz}$
- (d) None of these

(1.9) If $F\{F(t)\} = f(p)$, then the Shifting property of Fourier transform is: (CO5,K1) 1

- (a) $F\{F(x-a)\} = e^{iap} f(p)$
- (b) $F\{F(x+a)\} = e^{iap} f(p)$
- (c) $F\{F(ax)\} = e^{iap} f(p)$
- (d) None of these

(1.10) The Fourier sine transform of $F(x) = e^{-ax}$ is: (CO5, K1)

1

- (a) $\frac{a}{a^2 + b^2}$
 (b) $\frac{a}{a^2 - b^2}$
 (c) $\frac{p}{a^2 + p^2}$
 (d) $\frac{p}{a^2 - p^2}$

2. Attempt all parts:-

- (2.1) Find the particular integral of the differential equation $(D^2 - 3D + 2)y = e^{3x}$. (CO1, K2) 2
 (2.2) Find the fourier coefficient a_0 for $f(x) = x^2 - 2$, $-2 < x < 2$. (CO2, K3) 2
 (2.3) Find Laplace transform of the function $F(t) = \cos h at - \cos at$. (CO3, k2) 2
 (2.4) Solve the partial differential equation $yzp + zxq = xy$ (CO4, K1) 2
 (2.5) Write Convolution Theorem for Fourier Transform (CO5, K2) 2

SECTION-B

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Answer any one of the following:-

- (3.1) Solve the differential equation: $(D - 1)^2 y = xe^x \cos x$ (CO1, K3) 6
 (3.2) Solve: $\frac{dx}{dt} = 3x + 2y$, $\frac{dy}{dt} = 5x + 3y$ (CO1, K2) 6

Answer any one of the following:-

- (3.3) Test the convergence of the series $\frac{\sqrt{1}}{1 + \sqrt{1}} + \frac{\sqrt{2}}{2 + \sqrt{2}} + \frac{\sqrt{3}}{3 + \sqrt{3}} + \dots$ (CO2, K3) 6

- (3.4) Express $\sin x$ as a cosine series in the interval $0 < x < \pi$. (CO2, K3) 6

Answer any one of the following:-

- (3.5) Evaluate the integral $\int_0^\infty e^{-3t} \cdot t \cdot \sin t dt$. (CO3, K2) 6
 (3.6) Find the inverse Laplace of the function $f(s) = \log\left(\frac{s+1}{s-1}\right)$. (CO 3, k2) 6

Answer any one of the following:-

- (3.7) Solve the linear partial differential equation $(D^2 + DD' + D' - 1)z = \cos(x + 2y)$. (CO4, K3) 6
 (3.8) Solve the differential equation $x^2(y - z)p + y^2(z - x)q = z^2(x - y)$. (CO4, K2) 6

Answer any one of the following:-

- (3.9) Find the Fourier Cosine Transform of $\frac{1}{1 + x^2}$. (CO5, K2) 6
 (3.10) Find the Fourier transform of the following function 6

$$f(x) = \begin{cases} \frac{1}{2\varepsilon}, & \text{for } |x| \leq \varepsilon \\ 0, & \text{for } x > \varepsilon \end{cases} \quad (\text{CO5,K3})$$

SECTION-C

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4. Answer any one of the following:-

(4.1) Solve by the method of variation of parameters: $(D^2 + a^2)y = \sec ax$ (CO1, K3) 10

(4.2) Solve by changing the independent variable: 10

$$\cos x \frac{d^2 y}{dx^2} + \sin x \frac{dy}{dx} - 2y \cos^3 x = 2 \cos^5 x. \quad (\text{CO1,K3})$$

5. Answer any one of the following:-

(5.1) Test the convergence of the series $x + \frac{1 \cdot x^3}{2 \cdot 3} + \frac{1 \cdot 3 \cdot x^5}{2 \cdot 4 \cdot 5} + \frac{1 \cdot 3 \cdot 5 \cdot x^7}{2 \cdot 4 \cdot 6 \cdot 7} + \dots$ 10
 . (CO2,K3)

(5.2) Obtain the fourier series for the function $f(x) = x + x^2$ in the interval $-\pi < x < \pi$. 10
 Hence show that
 $\frac{\pi^2}{6} = 1 + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \dots$.(CO2,K3)

6. Answer any one of the following:-

(6.1) Using Convolution Theorem evaluate 10

$$L^{-1} \left\{ \frac{s^2}{(s^2 + 1)(s^2 + 4)} \right\}. \quad (\text{CO3,K2})$$

(6.2) Solve the following differential equation by using Laplace transformation 10
 $\frac{d^2 x}{dt^2} + 2 \frac{dx}{dt} + x = t e^{-t}$, Given that $x(0) = 1$, $x'(0) = 2$. (CO3,K3)

7. Answer any one of the following:-

(7.1) Solve $(D^2 - DD' + D' - 1)Z = \cos(x + 2y) + e^y$ (CO4,K3) 10

(7.2) Solve the linear partial differential equation $\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial x \partial y} - 6 \frac{\partial^2 z}{\partial y^2} = y \sin x$ (CO4, K3) 10

8. Answer any one of the following:-

(8.1) Find the Fourier cosine and Fourier sine transform of the function 10
 $f(x) = e^{-3x} + e^{-4x}$ (CO5,K3)

(8.2) Using Fourier sine integral, show that 10

$$\int_0^\infty \left(\frac{1 - \cos \pi \lambda}{\lambda} \right) \sin(x \lambda) d\lambda = \begin{cases} \pi/2, & \text{when } 0 < x < \pi \\ 0, & \text{when } x > \pi \end{cases} \quad (\text{CO5,K3})$$