Printed Page:-4 **Subject Code:- CCSBS0205**

Roll. No:

NOIDA INSTITUTE OF ENGINEERING AND TECHNOLOGY, GREATER NOIDA

(An Autonomous Institute Affiliated to AKTU, Lucknow)

B.Tech

SEM: II - THEORY EXAMINATION (2025 - 2026)

Subject: Linear Algebra

Time: 3 Hours **Max. Marks: 100**

General Instructions:

IMP: *Verify that you have received the question paper with the correct course, code, branch etc.*

1. *This Question paper comprises of* ***three Sections -A, B, & C.*** *It consists of Multiple Choice Questions (MCQ's) & Subjective type questions.*

2. *Maximum marks for each question are indicated on right -hand side of each question.*

3. *Illustrate your answers with neat sketches wherever necessary.*

4. *Assume suitable data if necessary.*

5. *Preferably, write the answers in sequential order.*

6. *No sheet should be left blank. Any written material after a blank sheet will not be evaluated/checked.*

SECTION-A 20

1. Attempt all parts:-

(1.1) The value of determinant $\begin{vmatrix} 1 & a & b+c \\ 1 & b & c+a \\ 1 & c & a+b \end{vmatrix}$ (CO1,K2) 1

(a) 0

(b) 1

(c) a+b+c

(d) 3

(1.2) If a matrix A is symmetric as well as skew-symmetric, then: (CO1, K1) 1

(a) A is diagonal matrix

(b) A is a unit matrix

(c) A is a triangular matrix

(d) A is a null matrix

(1.3) If $\rho(A) = \rho(A, B)$ then the system is (CO2,K1) 1

(a) Consistent and has infinitely many solutions

(b) Consistent and has a unique solution

(c) Consistent

(d) inconsistent

(1.4) A linear system is called consistent if it has (CO2,K1) 1

(a) infinitely many solutions

(b) at least one solution

(c) no solution

(d) none of the above

(1.5) In the vector space V if $a + b = b + a$;where $a, b \in V$ is called (CO3,K1) 1

(a) Associativity
(b) Additive inverse
(c) Commutative
(d) None of these

(1.6) If *A* is a matrix with *n* columns, then (CO3,K2) 1

(a) $\text{rank}(A) + \text{nullity}(A) = n$
(b) $\text{rank}(A) - \text{nullity}(A) = n$
(c) $\text{rank}(A) + \text{nullity}(A) = 2n$
(d) $\text{rank}(A) - \text{nullity}(A) = n/2$

(1.7) If *u* and *v* are vectors in a real inner product space *V*, then (CO4,K1) 1

(a) $|\langle u,v\rangle| \geq \|u\| + \|v\|$
(b) $|\langle u,v\rangle| \leq \|u\| + \|v\|$
(c) $|\langle u,v\rangle| \geq \|u\| \ \|v\|$
(d) $|\langle u,v\rangle| \leq \|u\| \ \|v\|$

(1.8) What should be the Eigen values for the matrix given below? 1

$A = \begin{bmatrix} 6 & 3 \\ 4 & 5 \end{bmatrix}$ (CO4,K3)

(a) 2 & 9
(b) 2 & 10
(c) 1 & 9
(d) 2 & 7

(1.9) Let *A* be a 2×2 real matrix with $\det(A) = 6$ and $\text{Trace}(A) = 4$. What is the value of $\text{Trace}(A^2)$? (CO5,K2) 1

(a) 10
(b) 9
(c) 7
(d) 4

(1.10) What is the role of the inverse of a matrix in solving linear equations? (CO5,K1) 1

(a) To represent the data in matrix format.
(b) To multiply the data with the weights.
(c) To find the solution to a system of linear equations.
(d) To calculate the eigenvalues of the matrix.

2. Attempt all parts:-

(2.1) Find the value of x if $\begin{vmatrix} 2 & 4 \\ 5 & 1 \end{vmatrix} = \begin{vmatrix} 2x & 4 \\ 6 & x \end{vmatrix}$. (CO1,K1) 2

(2.2) Find the rank of the matrix: 2

$A=\begin{bmatrix}1 & 4\\ 2 & 8\end{bmatrix}$. (CO2,K2)

(2.3) Define left and right distribution laws. (CO3,K1) 2

(2.4) If Eigen values of matrix A are 2, -3 & 1, find the eigen values of A^2 and A^{-1} . (CO4,K2) 2

(2.5) Find the singular values of matrix $\begin{bmatrix}1 & 3\\ 3 & 1\end{bmatrix}$.(CO5,K2) 2

SECTION-B 30

Answer any <u>one</u> of the following:-

(3.1) If $A=\begin{bmatrix}1 & 4 & 2\\ 0 & 5 & 1\\ 2 & 6 & 1\end{bmatrix}$ and $B=\begin{bmatrix}2 & 4 & 1\\ 5 & 1 & 4\\ -8 & 7 & 2\end{bmatrix}$, then find *AB* and *BA*. *(CO1,K2)* 6

(3.2) Find the adjoint and the inverse of the matrix $\begin{bmatrix}2 & 4 & 3\\ 0 & 1 & 1\\ 2 & 2 & -1\end{bmatrix}$. (CO1,K2) 6

Answer any one of the following:-

(3.3) Test the consistency of the system of equation: $x-2y+4z=2,\ 2x-3y+5z=3,\ 3x-4y+6z=7.$ (CO2,K2) 6

(3.4) If the vectors $(1,-1,2),(7,3,x)$ and $(2,3,1)$ in R^3 are linearly independent, then find the value of x. (CO2,K3) 6

Answer any one of the following:-

(3.5) If α *and* β are vector in real vector space and if $\alpha+\beta$ is orthogonal to α-β, then prove that $\|\alpha\|=\|\beta\|$. (CO3,K2) 6

(3.6) Let $v_1=(1,-1,0), v_2=(0,1,-1), v_3=(0,2,1)$ and $v_4=(1,0,3)$ be elements of R^3. Show that the set of vectors $\{v_1,v_2,v_3,v_4\}$ is linearly dependent. (CO3,K3) 6

Answer any one of the following:-

(3.7) Show that the mapping $T:V_3(R)\rightarrow V_2(R)$ defined as $T(a,b,c)=(a,b)$ is a linear transformation. (CO4,K3) 6

(3.8) Show that the matrix A is Skew- Hermitian and also Unitary $A=\begin{bmatrix}i & 0 & 0\\ 0 & 0 & i\\ 0 & i & 0\end{bmatrix}$. (CO4,K2) 6

Answer any one of the following:-

(3.9) Find the Eigen values and Eigen Vector of A^TA, where $A=\begin{bmatrix}2 & -1\\ 2 & 2\end{bmatrix}$. (CO5,K3) 6

(3.10) Write five applications of Singular Value Decomposition (SVD). (CO5,K2) 6

SECTION-C 50

4. Answer any <u>one</u> of the following:-

(4.1) Find the inverse of a matrix $A=\begin{bmatrix} 1 & -3 & 2 \\ 2 & 0 & 0 \\ 1 & 4 & 1 \end{bmatrix}$ by elementary transformation. (CO1,K2) 10

(4.2) Solve the system of equations for the linear equations with 3 variables using Cramer's rule. (CO1,K2) 10

$x - z = 11$

$7x - 4y - z = -15$

$4x + 6y + 5z = -6$

5. Answer any <u>one</u> of the following:-

(5.1) Solve the following equations by LU decomposition method. 10

$6x_1 + 18x_2 + 3x_3 = 3,\ 2x_1 + 12x_2 + x_3 = 19,\ 4x_1 + 15x_2 + 3x_3 = 0$ (CO2, K3)

(5.2) Find the rank of the following matrices by using elementary row transformations: 10

$A=\begin{bmatrix} 1 & -3 & 1 & 2 \\ 0 & 1 & 2 & 3 \\ 3 & 4 & 1 & -2 \end{bmatrix}$. (CO2,K2)

6. Answer any <u>one</u> of the following:-

(6.1) Define inner product space. Then show that $u=(u_1,u_2), v=(v_1,v_2)$ in R^2 defined by $\langle u,v\rangle = 4u_1v_1 + 5u_2v_2$ is inner product space. (CO3,K4) 10

(6.2) Prove that the set of all solutions (a, b, c) of the equation $a+b+2c = 0$ is a subspace of the vector space *V*. (CO3,K2) 10

7. Answer any <u>one</u> of the following:-

(7.1) Show that the matrix $\begin{bmatrix} \alpha+i\gamma & -\beta+i\delta \\ \beta+i\delta & \alpha-i\gamma \end{bmatrix}$ is unitary if $\alpha^2+\beta^2+\gamma^2+\delta^2 = 1$. (CO4,K3) 10

(7.2) Show that the mapping $T:V_2 \to V_3$ defined as $T(a,b) = (a+b, a-b, b)$ is a linear transformation from V_2 into V_3. Find the range, rank, null-space and nullity of *T*. (CO4,K3) 10

8. Answer any <u>one</u> of the following:-

(8.1) Find the covariance matrix and the principal components of the following: (CO5,K4) 10

X	2.5	0.5	2.2	1.9	3.1	2.3
Y	2.4	0.7	2.9	2.2	3.0	2.7

(8.2) Find the singular value decomposition of the matrix: $A=\begin{bmatrix} 3 & 1 & 1 \\ -1 & 3 & 1 \end{bmatrix}$.(CO5,K3) 10