Printed page: 4 **Subject Code:CMIAS0203A**

Roll No:

NOIDA INSTITUTE OF ENGINEERING AND TECHNOLOGY, GREATER NOIDA

(An Autonomous Institute Affiliated to AKTU, Lucknow)

M.Tech Integrated

SEM II THEORYEXAMINATION 2025-2026

Subject: Differential Equations and Linear Transformations

Time: 3 Hours **Max. Marks:100**

General Instructions:

IMP: Verify that you have received question paper with correct course, code, branch etc.

1. *This Question paper comprises of three Sections -A, B, & C. It consists of Multiple Choice Questions (MCQ's) & Subjective type questions.*
2. *Maximum marks for each question are indicated on right hand side of each question.*
3. *Illustrate your answers with neat sketches wherever necessary.*
4. *Assume suitable data if necessary.*
5. *Preferably, write the answers in sequential order.*
6. *No sheet should be left blank. Any written material after a blank sheet will not be evaluated/checked.*

SECTION – A 20

1. Attempt all parts:-

(1.1) The Complementary function of the differential equation $(D-2)^3 = 0$ is c : (CO1,K2) 1

(a) $(a + bx + cx^2)e^{2x}$

(b) $(a + bx - cx^2)e^{2x}$

(c) $(a - bx + cx^2)e^{2x}$

(d) None of these

(1.2) The differential equation $\frac{d^4y}{dx^4} + 13\frac{d^2y}{dx^2} + 36y = 0$ is : (CO1,K1) 1

(a) Order 4 and degree 1

(b) Order 4 and degree 2

(c) Order 1 and degree 4

(d) None of these

(1.3) The functions $\sin nx$ and $cos\ nx$ are periodic with period : (CO2,K1) 1

(a) $2\pi/n$

(b) π/n

(c) $x\pi/n$

(d) None of these

(1.4) The general half range sine series of function $f(x)$ in the $(0, l)$ can be given as : 1

(a) $f(x) = \sum_1^\infty b_n \, sin\frac{n\pi x}{l}$ (CO2, K1)

(b) $f(x) = \sum_1^\infty b_n \sin nx$

(c) $f(x) = \sum_1^\infty b_n \, sin\frac{\pi x}{l}$

(d) None of these

(1.5) If Laplace transform of $\{F(t)\} = f(s), \; then \; L\{F(at)\}$ is (CO3,K1) 1

(a) $1/a \; f(s/a)$

(b) $\frac{1}{a} \, f(s-a)$

(c) $f(\frac{s}{a})$

(d None of these

(1.6) Laplace transform of $f(t) = 5e^{2t} + 2$ is : (CO3,K2) 1

(a) $\frac{5}{s-2} - \frac{1}{s}$

(b) $\frac{1}{s-2}$

(c) $\frac{5}{s-2} + \frac{2}{s}$

(d) $\frac{1}{s-2} + \frac{1}{s}$

(1.7) The vectors $(1, 0, 0)(0,1,-1)$ and $(0, 1, 2)$ are: (CO4,K2) 1

(a) Linearly dependent

(b) Linearly independent

(c) (a) & (b) both

(d) None of these

(1.8) Which of the following does not form a basis for R^2 ? (CO4, K2) 1

(a) $(1,0), (0,1)$

(b) $(1,1), (2,2)$

(c) $(1,2), (3,4)$

(d) $(5,0), (0,3)$

(1.9) Let $T: R^5 \rightarrow R^3$ be a linear transformation. If the rank of is 2, what is the nullity of T ? (CO5, K2) 1

(a) 1
(b) 2
(c) 3
(d) None of these

(1.10) If α, β are two orthogonal unit vectors in an inner product space $V(F)$, then $\|\alpha + \beta\|$ is: (CO5, K2) 1

(a) 0

(b) $\sqrt{2}$

(c) 1

(d) None of these

2. Attempt all parts:-

(2.1)	Find the P.I. of differential equation $y'' + a^2y = sinax$. (CO1,K2)		2
(2.2)	Find the coefficient a_0 in a Fourier series for the Function $f(x) = x$ in the interval $0 < x < 2\pi$. (CO2,K3)		2
(2.3)	Find value of the integral $\int_0^\infty \frac{sint}{t} dt$. (CO3, K3)		2
(2.4)	Define Basis and Dimension of a Vector space. (CO4,K1)		2
(2.5)	Define Rank and Nullity theorem. (CO5,K1)		2

SECTION – B 30

Answer any <u>one</u> of the following-

(3.1) Solve the simultaneous differential equation $\frac{dx}{dt} + 5x - 2y = t, \frac{dy}{dt} + 2x + y = 0$. (CO1,K3) 6

(3.2) Solve the ordinary differential equation: $x^2 \frac{d^2y}{dx^2} - 4x \frac{dy}{dx} + 6y = x^2$. (CO1,K3) 6

Answer any <u>one</u> of the following-

(3.3) Solve the PDE : $x(y^2 + z)p - y(x^2 + z)q = z(x^2 - y^2)$. (CO2,K3) 6

(3.4) Find the half range cosine series expansion of $f(x) = x - x^2 in\ 0 < x < 1$. (CO2,K3) 6

Answer any <u>one</u> of the following-

(3.5) Express the following function in terms of unit step function and Find the Laplace transform of the function: $f(t) = \begin{cases} 2, & if\ 0 < t < \pi \\ 0, & if\ \pi < t < 2\pi \\ sint, & if\ t > 2\pi \end{cases}$ (CO3,K3) 6

(3.6) Find the Laplace Transform of $t^2e^t sint$. (CO3, K3) 6

Answer any <u>one</u> of the following-

(3.7) If $V = \{(x, y, z): x, y, z \in R\}$ where R is the field of real numbers then show that the set $W = \{(x, y, z): x - 3y + 4z = 0\}$ is a subspace of V over R . (CO4, K3) 6

(3.8) Determine a basis and dimension of the subspace spanned by the vectors $a_1 = (1,0,0), a_2 = (0,2,0), a_3 = (0,0,3)$ and $a_4 = (1,2,3)$. (CO4, K3) 6

Answer any <u>one</u> of the following-

(3.9) Let $T : V_2(R) \rightarrow V_2(R)$ is defined by $T(x, y) = (3x + 2y, 3x - 4y)$. Verify whether T is a linear transformation. Find its rank and nullity. (CO5, K3) 6

(3.10) Describe explicitly a linear transformation from $R^3 \rightarrow R^3$ which has its range and subspace spanned by the vectors (1,0,-1) and (1,2,2). (CO5, K3) 6

SECTION – C 50

4. Answer any <u>one</u> of the following-

(4.1) Solve the following differential equation by variation of parameter method : $\frac{d^2y}{dx^2} + y = tanx$. (CO1,K3) 10

(4.2) Solve the ODE by changing independent variable $x\frac{d^2y}{dx^2} - \frac{dy}{dx} + 4x^3y = x^5$. (CO1, K3) 10

5. Answer any <u>one</u> of the following-

(5.1) Find the Fourier series for $f(x) = x + x^2, -\pi < x < \pi$ Hence deduce that $\frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} \dots\dots\dots = \frac{\pi^2}{12}$. (CO2,K3) 10

(5.2) Solve the linear partial differential equation $\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial x \partial y} - 2\frac{\partial^2 z}{\partial y^2} = (y-1)e^x$ (CO2,K3) 10

6. Answer any <u>one</u> of the following-

(6.1) Solve the differential equation by Laplace transform $\frac{d^2y}{dt^2} - 3\frac{dy}{dt} + 2y = 4t + e^{3t}$ given that $y(0) = 1, \frac{dy}{dt} = -1$ for $t = 0$. (CO3,K3) 10

(6.2) Apply Convolution theorem to evaluate $L^{-1}\left\{\frac{s^2}{(s^2+1)(s^2+4)}\right\}$. (CO3,K3) 10

7. Answer any <u>one</u> of the following-

(7.1) Let V be the set of all pairs (x, y) of real numbers, and let F be the field of real numbers. Define the addition and scalar multiplication in $(x_1, y_1) + (x_2, y_2) = (x_1 + x_2, y_1 + y_2)$ and $a(x, y) = (ax, ay)$. Is V, with these operations, a vector space over the field of real numbers? (CO4, K3) 10

(7.2) Are the following vectors linearly dependent? If so find the relation between them: $X_1 = (1,2,1), X_2 = (2,1,4), X_3 = (4,5,6)$ and $X_4 = (1,8,-1)$. (CO4,K3) 10

8. Answer any <u>one</u> of the following-

(8.1) Apply Gram-Schmidt Process to the following sequence of vectors to obtain an orthonormal basis for $R^3(R)$ with the standard inner product : $\begin{bmatrix}1\\2\\0\end{bmatrix} \begin{bmatrix}8\\1\\-6\end{bmatrix} \begin{bmatrix}0\\0\\1\end{bmatrix}$. (CO5,K3) 10

(8.2) If α and β are vectors in an inner product space $V(F)$, and $a, b \in F$ then prove that $Re(\alpha, \beta) = \frac{1}{4}\|\alpha + \beta\|^2 - \frac{1}{4}\|\alpha - \beta\|^2$. (CO5,K3) 10