

--	--	--	--	--	--	--	--	--	--	--	--	--	--	--

NOIDA INSTITUTE OF ENGINEERING AND TECHNOLOGY, GREATER NOIDA
(An Autonomous Institute Affiliated to AKTU, Lucknow)

B.Tech

SEM: IV - THEORY EXAMINATION (2025 - 2026)

Subject: Probability, Statistics and Complex Analysis

Time: 3 Hours

Max. Marks: 100

General Instructions:

IMP: Verify that you have received the question paper with the correct course, code, branch etc.

1. This Question paper comprises of **three Sections -A, B, & C**. It consists of Multiple Choice Questions (MCQ's) & Subjective type questions.

2. Maximum marks for each question are indicated on right -hand side of each question.

3. Illustrate your answers with neat sketches wherever necessary.

4. Assume suitable data if necessary.

5. Preferably, write the answers in sequential order.

6. No sheet should be left blank. Any written material after a blank sheet will not be evaluated/checked.

SECTION-A

20

1. Attempt all parts:-

- (1.1) In a frequency curve of scores, the mode was found to be higher than the mean. This shows that the distribution is: (CO1, K2) 1
- (a) Negatively skewed
- (b) Symmetric
- (c) Positively skewed
- (d) None of the above
- (1.2) The standard deviation of a symmetric distribution is 5 then the value of the fourth moment about mean in order that the distribution will be leptokurtic is: (CO1, K3) 1
- (a) $\mu_4 > 1875$
- (b) $\mu_4 < 1875$
- (c) $\mu_4 = 1875$
- (d) $\mu_4 > 1888$
- (1.3) If two dice are thrown, then total number of outcomes is: (CO2,K2) 1
- (a) 6
- (b) 36
- (c) 12
- (d) 8
- (1.4) Mathematical probability is defined as: (CO2,K1) 1
- (a) Favorable outcomes / Total outcomes
- (b) Total outcomes / Favorable outcomes
- (c) Difference of outcomes
- (d) Product of outcomes

- (1.5) For Binomial distribution the sum and difference of mean and variance are 6 and 2 respectively, then the probability of success in single trial is: (CO3, K3) 1
- (a) 1
(b) 0.5
(c) 0.25
(d) 0.75
- (1.6) Normal Distribution is symmetric about: (CO3,K1) 1
- (a) Variance
(b) Mean
(c) Standard deviation
(d) Covariance
- (1.7) If $u(x, y) = 2x(1 - y)$ is given then: (CO4,K3) 1
- (a) $v = y^2 - x^2 + 3y + c$
(b) $v = y^2 - x^2 + 4y + c$
(c) $v = y^2 - x^2 + 6y + c$
(d) $v = -y^2 + x^2 + 2y + c$
- (1.8) The harmonic conjugate function of $v(x, y)$ of $u(x, y) = e^x \cos y$ is: (CO4, K3) 1
- (a) $e^x \sin y$
(b) $-e^x \sin y$
(c) 0
(d) $e^x \operatorname{cosec} y$
- (1.9) If C is closed curve $|Z| = r$ and $n \neq -1$, then the value of $\int_C z^n dz$ is: (CO5,K3) 1
- (a) $2\pi i$
(b) πi
(c) 1
(d) 0
- (1.10) Residue of $\frac{z^3}{z^2 - 1}$ at $z = \infty$ is: (CO5,K3) 1
- (a) ∞
(b) 1
(c) 0
(d) -1

2. Attempt all parts:-

- (2.1) The standard deviation of symmetric distribution is 4. Find the value of μ_4 so that the distribution must be mesokurtic. (CO1, K3) 2
- (2.2) A card is drawn from a well-shuffled deck of 52 cards. Find the probability that it is either a king or a queen. (CO2,K3) 2
- (2.3) A die is tossed twice. A success is getting 1 or 6 on a toss. Find the variance of the number of success. (CO3, K3) 2
- (2.4) Show that $u(x, y) = e^x \sin y$ is harmonic function. (CO4,K3) 2

(2.5) Define singularity of complex function. (CO5,K1) 2

SECTION-B 30

Answer any one of the following:-

(3.1) Calculate Quartile deviation and its coefficient from the following data: (CO1, K3) 6

C.I.	0-4	5-9	10-14	15-19	20-24	25-29
Freq.	15	26	12	5	4	3

(3.2) Calculate Bowley's coefficient of skewness from the data given below. (CO1, K3) 6

x	5	10	15	20	25	30	35	40	45
f	9	10	12	15	11	7	6	5	2

Answer any one of the following:-

(3.3) Two dice are thrown together. Find the probability that the sum of numbers is 7 or 11. (CO2,K3) 6

(3.4) A bag contains 5 white and 3 black balls. Two balls are drawn one after another without replacement. Find the probability that both are different colors. (CO2,K3) 6

Answer any one of the following:-

(3.5) It is given that 2% of the electric bulbs manufactured by a company are defective. Using Poisson distribution find the probability that a sample of 200 bulbs will contain: (i) No defective (ii) at least one defective bulb. (CO3, K3) 6

(3.6) Determine the minimum marks of a student must get in order to receive an A grade. If the top 10% of the students are awarded A grades in an examination where the mean marks is 72 and standard deviation is 9. Assume that the marks are normally distributed. Given that the area of standard normal curve from $z=0$ to $z=1.29$ is 0.4. (CO3, K3) 6

Answer any one of the following:-

(3.7) Show that the function $u(x,y) = 2x + y^3 - 3x^2y$ is harmonic. Find it's harmonic conjugate. (CO4,K3) 6

(3.8) If $u(x,y) = x^3y - xy^3$ is real part of an analytic function then find it's harmonic conjugate function $v(x,y)$. (CO4,K3) 6

Answer any one of the following:-

(3.9) Determine the poles and residue at each pole of the function $f(z) = \frac{z^2}{(z-2)(z-1)^2}$. (CO5,K3) 6

(3.10) Evaluate $\int_C \frac{(2z^2+5)}{(z+2)^3(z^2+4)} dz$, Where C is the square with vertices at $1+i$, $2+i$, $2+2i$, $1+2i$. (CO5,K3) 6

SECTION-C 50

4. Answer any one of the following:-

(4.1) Find the first four moments about the mean of the following distribution: 10

Height (in inches)	60-62	63-65	66-68	69-71	72-74
Frequency	5	18	42	27	8

Also compute β_1 and β_2 . (CO1, K3)

- (4.2) The first four moments of a distribution about the value '4' of the variable are -1.5, 17, -30 and 108. Find the moments about mean, about origin, β_1 and β_2 . Also, find the moments about the point $x=2$. (CO1, K3) 10

5. Answer any one of the following:-

- (5.1) Two dice are thrown together. Find the probability that the product of the numbers is 12 or the sum is 8. (CO2,K3) 10
- (5.2) From a pack of 52 cards, two card is drawn at random. Find the probability that both either a spade or a queen. (CO2,K3) 10

6. Answer any one of the following:-

- (6.1) Seven coins are tossed and no. of heads noted. The experiment is repeated 128 times and the following distribution is obtained: 10

no. of heads	0	1	2	3	4	5	6	7	total
frequencies	7	6	19	35	30	23	7	1	128

Fit a binomial distribution to the above data. (CO3, K3)

- (6.2) The probability distribution of random variable $X=r$ is given by $P(X=r) = q^{r-1}p$, Where $r=1,2,3..$, then find the Moment generating function of X . (CO3, K3) 10

7. Answer any one of the following:-

- (7.1) Define Harmonic function. Show that $v = \log(x^2 + y^2) + x - 2y$ is harmonic. Also find the analytic function $f(z)$. (CO4,K3) 10
- (7.2) Define analytic function. Discuss the analyticity of $f(z) = \operatorname{Re}(z^3)$. (CO4,K3) 10

8. Answer any one of the following:-

- (8.1) Verify Cauchy's integral theorem for $f(z) = 3z^2 + iz - 4$ along the boundary of the square with vertices $(1+i)$, $(1-i)$, $(-1+i)$, $(-1-i)$. (CO5,K3) 10
- (8.2) Use Cauchy's integral formula to evaluate $\int_C \frac{(5z-2)}{(z^2-z)} dz$, where C is the circle $|z| = 2$. (CO5,K3) 10