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NOIDA INSTITUTE OF ENGINEERING AND TECHNOLOGY, GREATER NOIDA
(An Autonomous Institute Affiliated to AKTU, Lucknow)

B.Tech

SEM: IV - THEORY EXAMINATION (2025 - 2026)

Subject: Advance Engineering Mathematics

Time: 3 Hours

Max. Marks: 100

General Instructions:

IMP: Verify that you have received the question paper with the correct course, code, branch etc.

1. This Question paper comprises of **three Sections -A, B, & C**. It consists of Multiple Choice Questions (MCQ's) & Subjective type questions.
2. Maximum marks for each question are indicated on right -hand side of each question.
3. Illustrate your answers with neat sketches wherever necessary.
4. Assume suitable data if necessary.
5. Preferably, write the answers in sequential order.
6. No sheet should be left blank. Any written material after a blank sheet will not be evaluated/checked.

SECTION-A

20

1. Attempt all parts:-

- (1.1) The mean of first n natural numbers is: (CO1,K3) 1
- (a) $(n+1)/2$
 - (b) $n/2$
 - (c) $n(n+1)/2$
 - (d) $(n-1)/2$
- (1.2) If the quartile deviation of a data set is zero, then the data is: (CO1, K2) 1
- (a) Data is uniformly distributed
 - (b) Indicating no spread in the middle 50% (Q_1 is equal to Q_3)
 - (c) Mean and Median are equal
 - (d) Data has extreme outliers
- (1.3) The MGF of the sum of two independent random variables x and y is: (CO2, K1) 1
- (a) $M_{x+y}(t) = M_x(t) + M_y(t)$
 - (b) $M_{x+y}(t) = M_x(t) \times M_y(t)$
 - (c) $M_{x+y}(t) = M_x(t) / M_y(t)$
 - (d) $M_{x+y}(t) = M_x(t) - M_y(t)$
- (1.4) If X is a random variable and a and b are constants then Expectation $E(aX+b)$ is: 1
- (CO2, K1)
- (a) $aE(x)+b$
 - (b) $E(x)+b$
 - (c) $aE(x)+0$
 - (d) $a^2E(x)+b$

- (1.5) The function $f(z) = \bar{z}$ is (CO3,K3) 1
- Analytic everywhere
 - Analytic nowhere
 - Analytic at origin
 - None of these
- (1.6) The functions f and g are analytic functions then (CO3, K1) 1
- f/g is analytic everywhere
 - f/g is analytic nowhere
 - f/g is analytic whenever $g(x)$ not equals to zero
 - f/g is analytic whenever $f(x)$ not equals to zero
- (1.7) $f(z) = \frac{z}{(z+3)(z-2)^3}$, poles and order of the poles are (CO4,K2) 1
- 3, 2 and 1, 3 respectively
 - 3, -2 and 1, 3 respectively
 - 3, 2 and 1, 1 respectively
 - None of these
- (1.8) For $f(z) = \frac{(z-1)}{(z+1)^2(z-2)}$, value of $\oint_C f(z)dz$, where C is the circle $|z| = 1/2$ (CO4, K3) 1
- 2π
 - 0
 - $2\pi i$
 - πi
- (1.9) If $F\{F(t)\} = f(p)$, then the Fourier transform $F(at)$ will be: (CO5,K1) 1
- $f\left(\frac{p}{a}\right)$
 - $\frac{1}{a} f\left(\frac{p}{a}\right)$
 - $a f\left(\frac{p}{a}\right)$
 - $\frac{1}{a} f(p)$
- (1.10) If Fourier transform of $F(x)$ is $f(p)$, then Fourier transform of $F(x+4)$ is: (CO5,K2) 1
- $e^{4ip} f(p)$
 - $e^{-4ip} f(p)$
 - $f(p+4)$
 - $4 f(p)$

2. Attempt all parts:-

- (2.1) If the sum of 20 observations is 300 and the sum of the squares of the observations is 5000 and median is 15. (CO1,K3) 2

Calculate Karl Pearson's coefficient of Skewness.

- (2.2) If a random variable x follows Poisson distribution such that $P(X=1)=P(X=2)$. find the mean. (CO2, K3) 2
- (2.3) If $f(z) = (u + iv)$ is an analytic function and $u = x^2 - y^2$, then find the conjugate harmonic function. (CO3, K3) 2
- (2.4) Evaluate: $\int_0^{(1+i)} (z^2) dz$, along the path $y = x$. (CO4, K3) 2
- (2.5) Find the fourier transform of the function $f(x) = \begin{cases} 1 & |x| < 1 \\ 0 & |x| > 1 \end{cases}$. (CO5, K3) 2

SECTION-B

Answer any one of the following:-

- (3.1) Calculate Quartile deviation and its coefficient from the following data: (CO1, K3) 6

C.I.	0-4	5-9	10-14	15-19	20-24	25-29
Freq.	15	26	12	5	4	3

- (3.2) Find the first four moments for the following frequency distribution about the mean: (CO1, K3) 6

x	0	1	2	3	4	5	6	7	8
f	1	8	28	56	70	56	28	8	1

Answer any one of the following:-

- (3.3) If 10% of the bolts produced by a machine are defective, determine the probability that out of 10 bolts chosen at random (i) none (ii) one (iii) at most 2 bolts will be defective. (CO2, K3) 6
- (3.4) In a test on 2000 electric bulbs, it was found that the life of a particular make, was normally distributed with an average life of 2040 hours and S.D. of 60 hours, estimate the number of bulbs likely to burn for More than 2150 hours. Given that the area of standard normal curve from $z=0$ to $z=1.83$ is 0.4664. (CO2, K3) 6

Answer any one of the following:-

- (3.5) Show that the function $u(x,y) = x^2 - y^2 - y$ is harmonic find harmonic conjugate function. (CO3, K3) 6
- (3.6) Define analytic function. Discuss the analyticity of z^3 . (CO3, K3) 6

Answer any one of the following:-

- (3.7) Evaluate $\oint_C \frac{e^z}{(z-1)(z-4)} dz$, where C is the circle $|z| = 2$. (CO4, K3) 6
- (3.8) Discuss the nature of singularity of $f(z) = \frac{(z - \sin z)}{z^3}$ at $z = 0$. (CO4, K2) 6

Answer any one of the following:-

- (3.9) Find the Fourier sine transform of $\frac{e^{-ax}}{x}$, $a > 0$ (CO5, K3) 6
- (3.10) Find the Fourier transform of the function $f(x) = \begin{cases} e^{inx} & a < x < b \\ 0 & \text{otherwise} \end{cases}$ (CO5, K3) 6

SECTION-C

4. Answer any one of the following:-

- (4.1) Find moment coefficient of skewness and coefficient of kurtosis of the frequency distribution given below and comment on the nature of distribution. (CO1, K3) 10

C.I.	20-30	30-40	40-50	50-60	60-70	70-80	80-90
Frequency	3	5	7	9	11	13	15

- (4.2) Obtain the median and mode of the following data. (CO1, K3) 10

No. of days absent	Below 5	Below 10	Below 15	Below 20	Below 25	Below 30	Below 35	Below 40	Below 45
No. of students	29	224	465	582	634	644	650	653	655

5. Answer any one of the following:-

- (5.1) Four coins were tossed 200 times. The number of tosses showing 0,1,2,3 and 4 heads were found to be as under. Fit a binomial distribution to these observed results. Find the expected frequencies. (CO2, K3) 10

No. of heads	0	1	2	3	4
No of tosses	15	35	90	40	20

- (5.2) If the pdf of random variable X is $f(x) = \begin{cases} kx^2(1-x), & 0 < x < 1 \\ 0, & \text{elsewhere} \end{cases}$. Calculate the (a) Value of k (b) Mean of X (c) $P(0.5 < X < 1.5)$. (CO2, K3) 10

6. Answer any one of the following:-

- (6.1) Examine the nature of the function $f(z) = \frac{x^2 y^5 (x+iy)}{(x^4+y^{10})}$ $z \neq 0$ and $f(z) = 0$ at $z = 0$ in the region including the origin. (CO3, K3) 10

- (6.2) Determine the analytic function whose imaginary part is $e^{-x}(x \cos y + y \sin y)$ (CO3, K3) 10

7. Answer any one of the following:-

- (7.1) Evaluate by using Cauchy's Residue theorem: $\int_C \frac{(4-3z)}{z(z-1)(z-2)} dz$, where C is the circle $|z| = 4$. (CO4, K3) 10

- (7.2) Find the Taylor's series expansion of $f(z) = \frac{(z+1)}{(z-3)(z-4)}$ about $z = 2$. (CO4, K3) 10

8. Answer any one of the following:-

- (8.1) Find the Fourier transform of $f(x) = \begin{cases} 1-x^2 & |x| < 1 \\ 0 & |x| > 1 \end{cases}$. Hence find 10

$$\int_0^\infty \left(\frac{x \cos x - \sin x}{x^3} \right) \cos \frac{x}{2} dx \quad (\text{CO5, K3})$$

- (8.2) Find the Fourier transform of the function $f(x) = \begin{cases} x^2 & |x| \leq a \\ 0 & x > a \end{cases}$. (CO5, K3) 10