Printed Page:-4 **Subject Code:- BASMC0401**

Roll. No:

NOIDA INSTITUTE OF ENGINEERING AND TECHNOLOGY, GREATER NOIDA

(An Autonomous Institute Affiliated to AKTU, Lucknow)

B.Tech

SEM: IV - THEORY EXAMINATION (2025 - 2026)

Subject: Applied Linear Algebra and Quantum Mechanics

Time: 3 Hours **Max. Marks: 100**

General Instructions:

IMP: *Verify that you have received the question paper with the correct course, code, branch etc.*

1. *This Question paper comprises of* ***three Sections -A, B, & C.*** *It consists of Multiple Choice Questions (MCQ's) & Subjective type questions.*

2. *Maximum marks for each question are indicated on right -hand side of each question.*

3. *Illustrate your answers with neat sketches wherever necessary.*

4. *Assume suitable data if necessary.*

5. *Preferably, write the answers in sequential order.*

6. *No sheet should be left blank. Any written material after a blank sheet will not be evaluated/checked.*

SECTION-A 20

1. Attempt all parts:-

(1.1) The set {(1,0),(0,1)} is a basis of: (CO1,K2) 1

(a) R^4

(b) R^2

(c) R^1

(d) R^3

(1.2) Following is a vector space over ℝ...(CO1,K2) 1

(a) Set of all 2×2 matrices under addition and scalar multiplication

(b) Set of all positive integers under addition

(c) Set of all polynomials of degree ≤ 3 under addition and scalar multiplication

(d) Both (A) and (C)

(1.3) If λ is an eigenvalue of A, then eigenvalue of A^2 is: (CO2,K2) 1

(a) 2λ

(b) λ

(c) 1/λ

(d) λ^2

(1.4) The following matrices is always diagonalizable.. (CO2,K2) 1

(a) Any square matrix

(b) Any symmetric matrix

(c) Any triangular matrix

(d) None of these

(1.5) The expectation value of an observable gives the ________ measurement result. (CO3, K3) 1

(a) average
(b) minimum
(c) maximum
(d) random

(1.6) The completeness relation in Hilbert space is related to ________. (CO3, K2) 1

(a) orthogonality
(b) boundary conditions
(c) basis expansion
(d) normalization only

(1.7) A mixed state is a statistical mixture of ________. (CO4, K2) 1

(a) operators
(b) eigenstates
(c) measurements
(d) gates

(1.8) Quantum entanglement refers to ________. (CO4, K2) 1

(a) independent states
(b) local copies
(c) separable joint states
(d) non-separable joint states

(1.9) Quantum parallelism allows simultaneous computation on __________. (CO5, K2) 1

(a) A qubit
(b) A classical bit
(c) Both
(d) All basis states

(1.10) Quantum circuits exploit interference by __________ states. (CO5, K3) 1

(a) Deleting
(b) Combining
(c) Ignoring
(d) Repeating

2. Attempt all parts:-

(2.1) Define Rank and nullity in vector Space. (CO1,K1) 2

(2.2) Write characteristics equation of given matrix $\mathbf{A}=\begin{bmatrix} 1 & 2 & 3 \\ 0 & -4 & 2 \\ 0 & 0 & 7 \end{bmatrix}$ (CO2,K3) 2

(2.3) Explain matrix elements in Dirac notation. (CO3. K2) 2

(2.4) Derive the position-momentum uncertainty relation. (CO4, K3) 2

(2.5) Explain the difference between classical and quantum gates using examples. (CO5, K2) 2

SECTION-B 30

Answer any <u>one</u> of the following:-

(3.1) Find the value of λ for which the vectors $(1,-2,\lambda)$, $(2,-1,5)$ and $(3,-5,7\lambda)$ are linearly dependent(CO1,K3) 6

(3.2) Find the basis and dimension of the subspace W of R^4 generated by (1, 4, -1, 3), (2, 1, -3, -1) and (0, 2, 1, -5). (CO1,K3) 6

Answer any one of the following:-

(3.3) Find the Eigen values and Eigen vectors of the following matrix $A=\begin{bmatrix} 2 & -1 \\ 5 & -2 \end{bmatrix}$ (CO2,K3) 6

(3.4) Reduce the quadratic $3x^2+5y^2+3z^2-2xy-2yz+2zx$ form to the canonical form and find the nature of the equation. (CO2,K3) 6

Answer any one of the following:-

(3.5) Discuss Hermitian operators in quantum mechanics. (CO3, K2) 6

(3.6) Describe the eigenvalue equation of an operator. (CO3, K2) 6

Answer any one of the following:-

(3.7) Examine the relation between density matrix and expectation values. (CO4, K4) 6

(3.8) Differentiate between separable and entangled states. (CO4, K4) 6

Answer any one of the following:-

(3.9) Explain the effect of applying a Hadamard gate on $|0\rangle$ and $|1\rangle$ and visualize on the Bloch sphere. (CO5, K2) 6

(3.10) Discuss quantum parallelism. Give an example of how it allows computation on multiple states simultaneously. (CO5, K2) 6

SECTION-C 50

4. Answer any <u>one</u> of the following:-

(4.1) Let $v=R^2=(x,y):x,y\in R$ and $F=R$. Define the addition and scalar multiplication in R^2 as follows $(x_1,y_1)+(x_2,y_2)=(x_1+x_2,y_1+y_2)$ and $a(x,y)=(ax,ay)$. Check R^2 is a vector space over R or not. (CO1,K3) 10

(4.2) Show that the set W of the elements of the vector space $V_3(R)$ of the form $(x,\ x,\ x)$ where $x\in R$ is a subspace of $V_3(R)$. (CO1,K3) 10

5. Answer any <u>one</u> of the following:-

(5.1) Find the eigen values of $3A^3+5A^2-6A+2I$ where $A=\begin{bmatrix} 1 & 2 & -3 \\ 0 & 3 & 2 \\ 0 & 0 & -2 \end{bmatrix}$. (CO2,K3) 10

(5.2) Find the eigenvalues and eigenvectors of the linear transformation $T:R^3\rightarrow R^3$ defined by $T(x,y,z)=(x+3z,\ 2x+y-z,\ x-y+z)$. (CO2,K3) 10

6. Answer any <u>one</u> of the following:-

(6.1) Examine how probabilities are obtained in quantum mechanics. (CO3, K4) 10

(6.2) Test orthogonality of quantum states. (CO3, K4) 10

7. Answer any <u>one</u> of the following:-

(7.1)	Relate the Born rule and its role in probability calculation. (CO4, K4)	10
(7.2)	Test how repeated measurements on the same system behave in quantum mechanics. (CO4, K4)	10

8. Answer any <u>one</u> of the following:-

(8.1)	Critically evaluate the difference between classical probabilistic mixtures and quantum superposition. Use Bloch sphere and/or circuit-based arguments to explain why interference is possible in the quantum case but not in a purely classical random mixture. (CO5, K5)	10
(8.2)	Consider a two-qubit entangled state, such as a Bell state. Explain how measurement of one qubit affects the state of the other. Support the state effect with mathematical derivation of post-measurement states for all possible measurement outcomes on the first qubit. (CO5, K5)	10